



High refractive index and lasing without inversion in an open four-level atomic system

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ABSTRACT

In this letter, a novel atomic scheme is proposed to study the transient evolution of the atomic response with applications to lasing with and without population inversion. We introduce an open four-level atomic medium and compare its transient properties with the corresponding closed system. The impact of cavity parameters i.e. the atomic exit rate from cavity and atomic injection rates on transient response of weak probe field of open system are investigated. It is realized that existence of cavity parameters leads to some interesting results such as large amplification, high refractive index without absorption as well as lasing with and without inversion. These results cannot be obtained in corresponding closed system, due to lack of atomic exit and injection rates. This extra controllability and flexibility, makes open four-level system much more practical than its counterpart closed one.

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1. Introduction

The optical properties of atomic gases can be radically modified by quantum coherence and quantum interference. Quantum coherence and interference in an atomic medium can result in many appealing outcomes. A marvellous consequence of preparing an atomic system in a coherent superposition of states is the absorption elimination that leads to the lasing without inversion [1,2], enhancement of the refraction index [3,4] and electromagnetically induced transparency (EIT) [5–9]. Under the conditions of electromagnetically induced transparency (EIT) it is feasible to control the optical response and related absorption of weak laser light. This effect has been deeply studied in atomic physics [10,11]. EIT has many noteworthy usages in quantum optics, such as the multi-wave mixing [12–16], enhancement of Kerr nonlinearity [17–20] and optical bistability and multistability [21]. More interestingly, the EIT effect has been found applications in quantum information science, such as the photon information storing and releasing in an atomic assemble [22], correlated photon pairs generation [23] and even the entanglement of remote atomic assembles [24], which form the building blocks of the quantum communication and the quantum computation. In view of many proposals, the transient properties of the weak probe field via

quantum interference such as transient-absorption, transient-dispersion, and transient-gain without inversion are widely investigated [24,25]. Zhu presented the condition required for observing the inversionless gain in the transient requirement for V [26] and Λ [27] schemes. The effect of SGC on transient process in the three-level system has also been investigated [28]. It is shown that EIT medium can be used as an absorptive optical switch [29], in which the transmission of highly absorptive medium is controlled dynamically by an additional signal (switching) light. Transient two-photon absorption property in a n-doped three-level semiconductor quantum well system is also investigated [30]. It is shown that the intensities and detunings of the optical fields can affect the two-photon absorption spectra dramatically, which can be used to suppress or enhance the two-photon absorption coefficient. Yang et al. [31] studied the transient and steady state absorption of a weak probe beam by means of a coupled double quantum well structure. However, almost all of these studies are considered with a closed system. An ideal level structure atomic system with appreciate interference and coherence features will bring great help to achieve more bright results. To the best of our knowledge, the transient properties of four-level open atomic media is never investigated, which motivates us to carry out this work. The presented scheme is based On Refs. [32,33], but our scheme is very different from those works. First, we investigate the transient evolution of the atomic response instead of steady-state response. Second, transient behavior in our scheme is realized by atomic exit rate and atomic injection rates which are characteristics of open

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systems and thus, is very different from other conventional closed schemes. Finally, we show new convenient ways to obtaining the high refractive index without absorption as well as lasing with and without absorption, which make our scheme much more practical than the other counterparts. Our paper is organized as follows: in Section 2, we present the model and equations. Numerical results and physical discussion are studied in Section 3. Section 4 presents some simple conclusions are given.

2. Model and equations

Fig. 1 denotes an open four-level atomic system coupled by a weak probe field, and two strong coupling fields. Levels $|1\rangle$ and $|3\rangle$ are coupled via a weak probe field with Rabi frequency of $\Omega_p = \vec{\varphi}_{31} \cdot \vec{E}_p / 2\hbar$ (amplitude E_p and frequency ω_p). A strong driving field with Rabi frequency of $\Omega_c = \vec{\varphi}_{32} \times \vec{E}_c / 2\hbar$ (amplitude E_c and frequency ω_c) is applied to transition $|2\rangle \rightarrow |3\rangle$. A coherent pump field of Rabi-frequency of $\Omega_s = \vec{\varphi}_{41} \times \vec{E}_s / 2\hbar$ with amplitude E_s and frequency ω_s couples levels $|1\rangle$ and $|4\rangle$. $\vec{\varphi}_{ij}$ denotes the corresponding electric dipole moment. The spontaneous decay rates from upper level $|4\rangle$ (and $|3\rangle$) to level $|1\rangle$ and $|2\rangle$ are defined as γ_{41}, γ_{42} (γ_{31}, γ_{32}), respectively. J_1 and J_2 are the atomic injection rates for levels $|1\rangle$ and $|2\rangle$, respectively. The ratio of the atomic injection rates is $X = J_2/J_1$. The atomic exit rate from the cavity is defined by r_0 . We also assume that the number of interacting atoms is constant, which means that $r_0 = J_1 + J_2$. Using the rotating-wave and the electric dipole

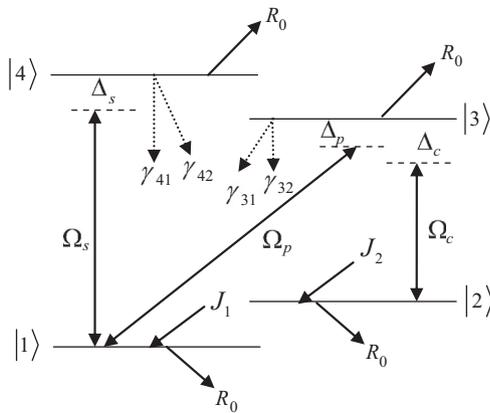


Fig. 1. The schematic of an open three-level ladder-type atomic system. The system will be a closed system if $r_0 = J_1 = J_2 = 0$.

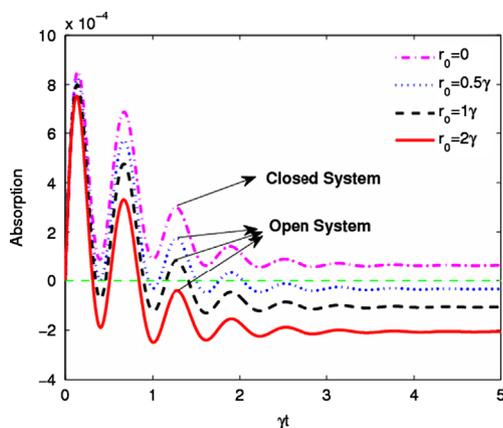


Fig. 2. Transient evolution of probe absorption for different values of r_0 . The parameters values are $\gamma_{31} = \gamma_{32} = \gamma_{42} = \gamma_{41} = 3\gamma$, $\Omega_c = \Omega_s = 5\gamma$, $\Delta_p = 0.8\gamma$, $\Delta_s = 0$, $\Delta_c = \gamma$, $\Omega_p = 0.01\gamma$.

approximations and in the interaction picture, the density matrix equations of motion of this system can be written as:

$$\begin{aligned} \dot{\rho}_{11} &= \gamma_{31}\rho_{33} + \gamma_{41}\rho_{44} + i\Omega_p(\rho_{31} - \rho_{13}) + i\Omega_s(\rho_{41} - \rho_{14}) + J_1 - r_0\rho_{11}, \\ \dot{\rho}_{22} &= \gamma_{32}\rho_{33} + \gamma_{42}\rho_{44} + i\Omega_c(\rho_{32} - \rho_{23}) + J_2 - r_0\rho_{22}, \\ \dot{\rho}_{33} &= -(\gamma_{31} + \gamma_{32})\rho_{33} + i\Omega_p(\rho_{13} - \rho_{31}) + i\Omega_c(\rho_{23} - \rho_{32}) - r_0\rho_{33}, \end{aligned}$$

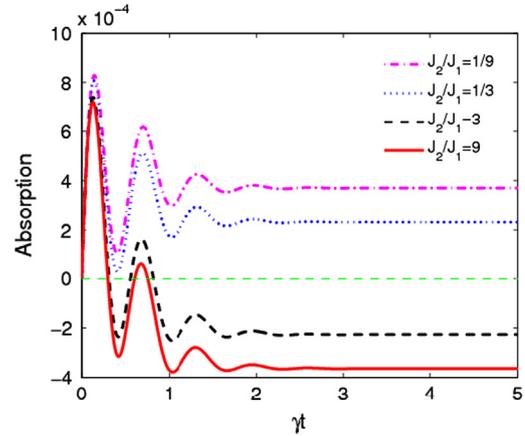


Fig. 3. Transient evolution of probe absorption for different values of X for open system. Here $r_0 = 2\gamma$ and the other parameters are the same as Fig. 2.

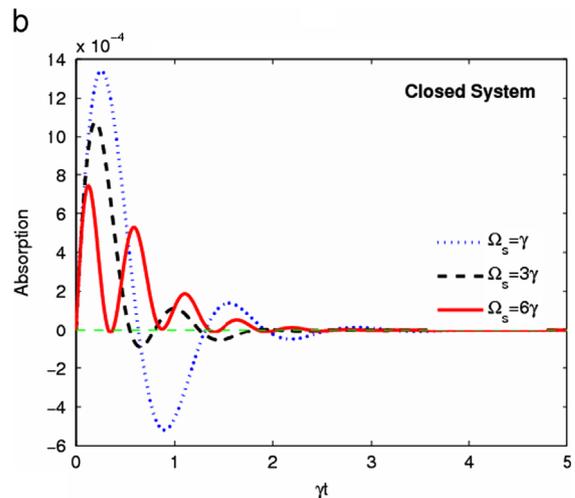
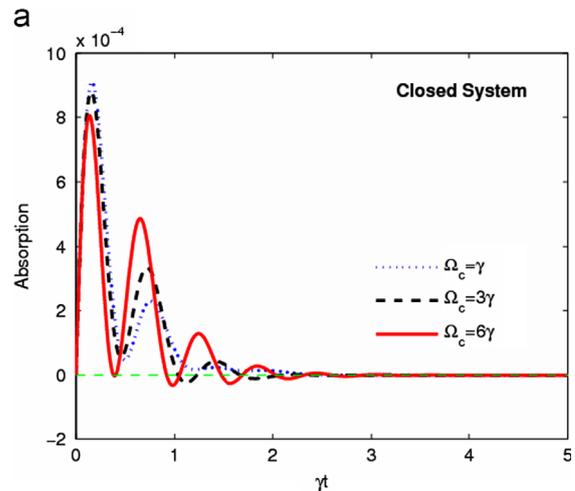


Fig. 4. Transient evolution of probe absorption for different values of (a) Ω_c , and (b) for closed system. The parameters values are (a) $\Omega_s = 5\gamma$ and (b) $\Omega_c = 5\gamma$, $r_0 = J_1 = J_2 = 0$, $\Delta_p = \Delta_s = \Delta_c = 0$, the other parameters are the same as Fig. 2.

$$\begin{aligned}
 \dot{\rho}_{12} &= i(\Delta_p - \Delta_c)\rho_{12} + i\Omega_p\rho_{32} + i\Omega_s\rho_{42} - i\Omega_c\rho_{13}, \\
 \dot{\rho}_{13} &= -\left[\frac{(\gamma_{31} + \gamma_{32})}{2} - i\Delta_p\right]\rho_{13} + i\Omega_s\rho_{43} - i\Omega_c\rho_{12} + i\Omega_p(\rho_{33} - \rho_{11}), \\
 \dot{\rho}_{14} &= -\left[\frac{(\gamma_{41} + \gamma_{42})}{2} - i\Delta_s\right]\rho_{14} + i\Omega_p\rho_{34} + i\Omega_s(\rho_{44} - \rho_{11}), \\
 \dot{\rho}_{23} &= -\left[\frac{(\gamma_{31} + \gamma_{32})}{2} - i\Delta_c\right]\rho_{23} - i\Omega_p\rho_{21} + i\Omega_c(\rho_{33} - \rho_{22}), \\
 \dot{\rho}_{24} &= -\left[\frac{(\gamma_{41} + \gamma_{42})}{2} + i(\Delta_p - \Delta_c - \Delta_s)\right]\rho_{24} + i\Omega_c\rho_{34} - i\Omega_s\rho_{21}, \\
 \dot{\rho}_{34} &= -\left[\frac{(\gamma_{31} + \gamma_{32} + \gamma_{41} + \gamma_{42})}{2} + i(\Delta_p - \Delta_s)\right]\rho_{34} + i\Omega_p\rho_{14} + i\Omega_c\rho_{24} - i\Omega_s\rho_{31}, \\
 \rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} &= 1.
 \end{aligned} \tag{1}$$

The frequency detuning parameters are defined as $\Delta_p = \omega_{31} - \omega_p$, $\Delta_c = \omega_{32} - \omega_c$, $\Delta_s = \omega_{41} - \omega_s$. In this set of equations, if $J_1 = J_2 = r_0 = 0$, Eq. (1) changes to those for a closed four-level atomic system [33].

3. Results and discussion

3.1. The analysis of the transient evolution of the atomic response

In the following, by using the numerical result from the density matrix equation of motions ρ_{ij} , we investigate the transient evolution of the atomic response from different respects. We are

interested in the effect of cavity parameters i.e. atomic injection rates and exit rate from cavity on transient properties of open four-level atomic scheme. As well known, gain-absorption and refractive index of the probe field on transition $|3\rangle \rightarrow |1\rangle$ are proportional to imaginary and real part of ρ_{31} which can be obtained from Eq. (1). If $\text{Im}(\rho_{31}) > 0$ the system exhibits absorption for the probe field, while for $\text{Im}(\rho_{31}) < 0$, the probe laser will be amplified. When $\rho_{33} > \rho_{11}$ and $\text{Im}(\rho_{31}) < 0$, the lasing with inversion can be obtained, whereas when $\rho_{33} < \rho_{11}$ and $\text{Im}(\rho_{31}) < 0$, the lasing without inversion can be realized. We now present the numerical results of Eq. (1) through Figs. 2–9. We assume $\gamma_{31} = \gamma_{32} = \gamma_{41} = \gamma_{42} = 3\gamma$ and all the figures are plotted in the unit of γ . Fig. 2 shows the transient evolution of the gain-absorption for various values of atomic exit rates in open system. Note that dash-dot curve in this figure shows the transient behavior of the corresponding closed system. We find that the absorption-gain curves have a oscillatory behavior for a short time and finally reach to the steady state as time increases. In addition, in closed system ($r_0 = J_1 = J_2 = 0$), the steady state value of absorption coefficient is positive relating to probe absorption, while for open system its value changes to negative that corresponds to a gain. Also, increasing the exit rate parameter to the larger values leads to the amplification enhancement. The impact of the ratio between atomic injection rates $X = J_2/J_1$ on transient behavior of the four-level open atomic system is plotted in Fig. 3. Obviously, for $X < 1$ probe absorption never manifests periodic gain and absorption; the absorption exhibits a oscillatory behavior in a short time and finally

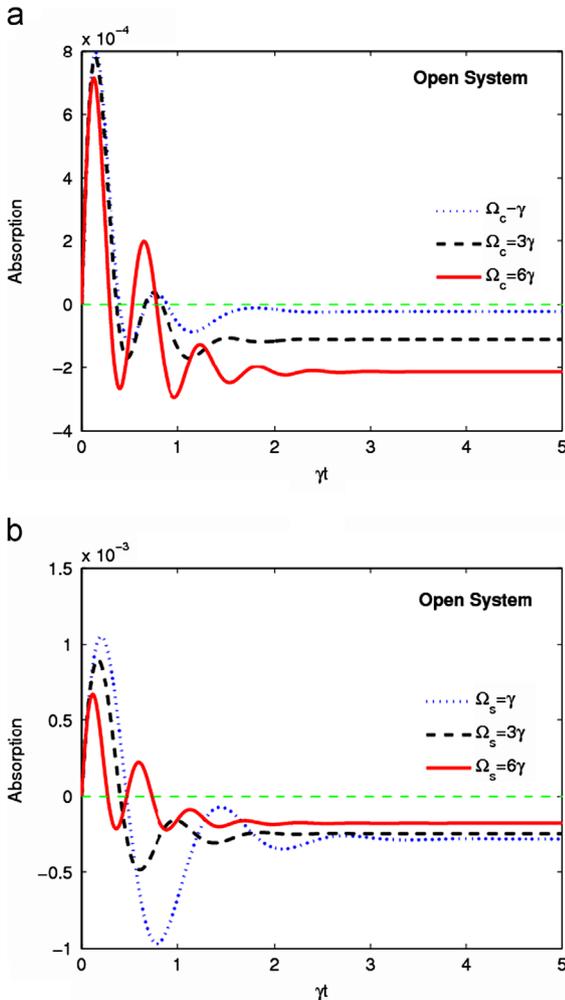


Fig. 5. Transient evolution of probe absorption for different values of (a) Ω_c , and Ω_s (b) for open system. The parameters values are (a) $\Omega_s = 5\gamma$ and (b) $\Omega_c = 5\gamma$, $r_0 = 2\gamma$, $X = J_2/J_1 = 3$, $\Delta_p = \Delta_s = \Delta_c = 0$, the other parameters are the same as Fig. 2.

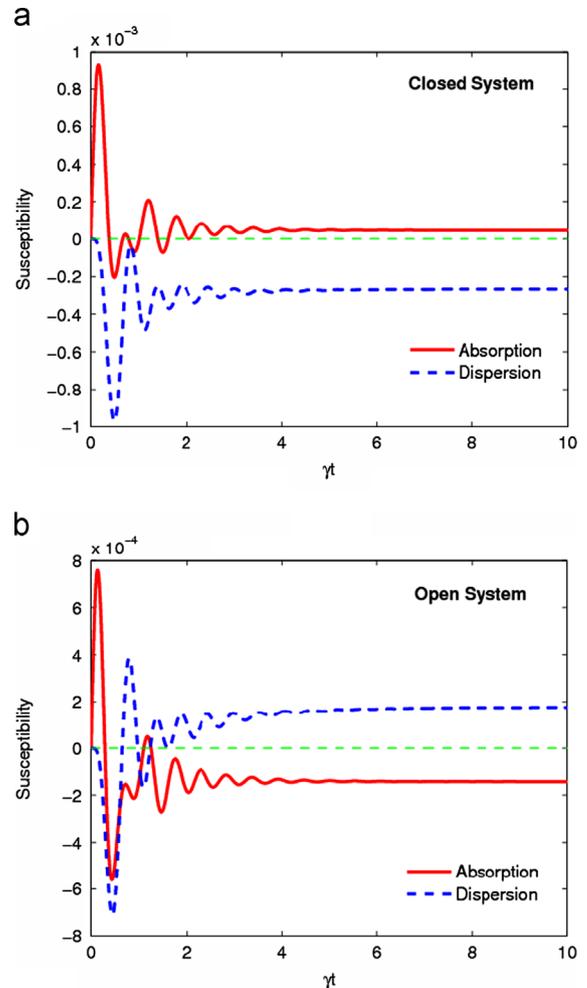


Fig. 6. Typical transient evolution of susceptibility (a) closed system ($r_0 = J_1 = J_2 = 0$), (b) open system ($r_0 = 2\gamma$, $X = J_2 = J_1 = 10$). The parameters values are $\Omega_c = 9\gamma$, $\Omega_s = \gamma$, $\Delta_p = 0$, $\Delta_c = 4\gamma$, $\Delta_s = 0.9\gamma$. The other parameters are the same as Fig. 2.

reaches to a positive steady state value. However, When $X > 1$, transient absorption finally disappears, only leaves transient gain below zero absorption line and reaches negative steady-state value. Thus, a non-periodic gain-absorption can be converted to a periodic one, just by increasing the atomic injection rate parameter X . In Fig. 4, we depicted the time evolution of the gain-absorption coefficient of different intensities of the driving fields Ω_c and Ω_s in closed (Fig. 4) and open (Fig. 5) systems. We observe different features of transient property for different values of Ω_c and Ω_s . In closed system (Fig. 4(a) and (b)), we can see that increasing the intensity of Rabi-frequencies Ω_c and Ω_s can affect the transient evolution process of probe absorption but doesn't affect steady state values. The steady state value for different intensities of Ω_c and Ω_s is near to zero and thus, electromagnetically induced transparency (EIT) occurs. In open system, investigation on Fig. 5(a) and (b) shows that by increasing Rabi-frequencies Ω_c and Ω_s , the oscillatory frequency of the gain-absorption coefficient curves increases, but the oscillatory amplitude decreases, and then it oscillates rapidly to a steady state value. Nevertheless, we can also distinguish between Fig. 5(a) and (b). Regarding to Fig. 5 (a) and (b), we deduce that the steady-state values of the amplification coefficient increases due to the increasing intensity of the Rabi-field Ω_c , whereas by increasing Ω_s the gain steady-state value reduces and comes close to zero absorption line. In other words, existence of atomic exit rate and injection rates make open system so sensitive on driving fields Ω_c and Ω_s and thus, has a critical rule to manipulate the transient behavior of system. The typical transient evolution of the susceptibility for closed (Fig. 6(a)) and open (Fig. 6(b)) system is

plotted for $\Omega_c = 9\gamma$, $\Omega_s = \gamma$ in closed system, it can be seen from Fig. 6 (a) that after an oscillating behavior, the absorption curve reaches to a positive steady state value corresponding to absorption. The refractive part of susceptibility is plotted here by dash-line. Clearly, by increasing the time, dispersion reaches to a negative at steady state. Transient behavior of real and imaginary part of susceptibility for corresponding open system ($r_0 = 2\gamma$, $X = J_2/J_1 = 10$) is depicted in Fig. 6(b). The results presents a completely oppositional transient behavior for $\text{Im}(\rho_{31})$ and $\text{Re}(\rho_{31})$. In this case, the steady state value of $\text{Im}(\rho_{31})$ has negative value which shows an amplification, but the steady state value for transient dispersion spectra becomes positive. Therefore, an advantage of open system than close system to create amplification instead of absorption and also to convert negative dispersion to positive dispersion or vice versa is presented which is practical in all- optical memories. Moreover, another interesting conclusion from Fig. 6 is presented here. An investigation on Fig. 6(a) shows that refractive index is always accompanied by a large absorption in closed system. But a high refractive index without absorption can be achieved in open system, as shown in Fig. 6(b). This condition can not be obtained in closed system.

3.2. Results for lasing with and without inversion

Now, we present details of our numerical results for lasing with and without inversion in Figs. 7–9. In the following numerical calculations we assume ($\Omega_c = 3\gamma$, $\Omega_s = \gamma$). For closed system, typical transient evolution of probe absorption (Fig. 7(a)) as well as

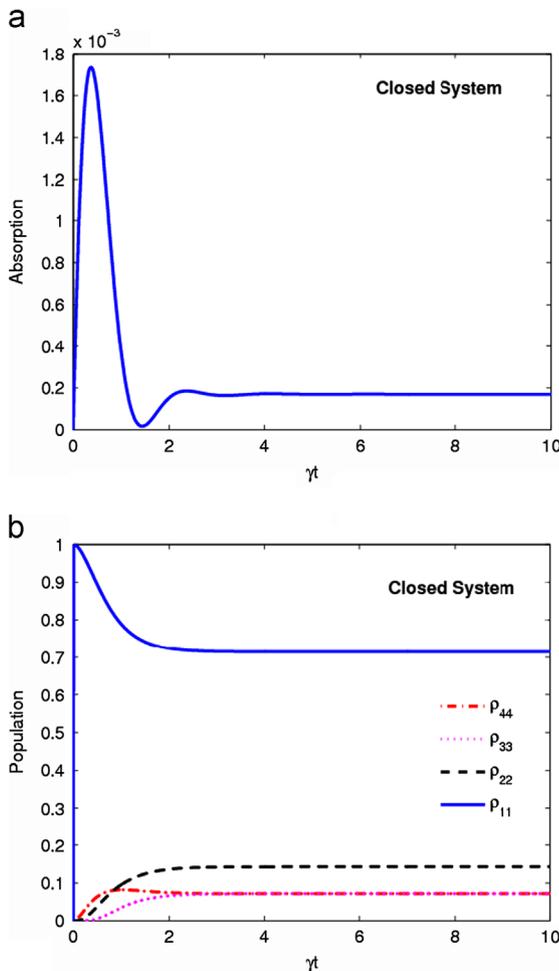


Fig. 7. Transient behaviors of (a) probe field absorption and (b) population distribution. The parameters values are $\Omega_c = 3\gamma$, $\Omega_s = \gamma$, $\Delta_p = \Delta_c = \Delta_s = 0$, $r_0 = J_1 = J_2 = 0$. The other parameters are the same as Fig. 2.

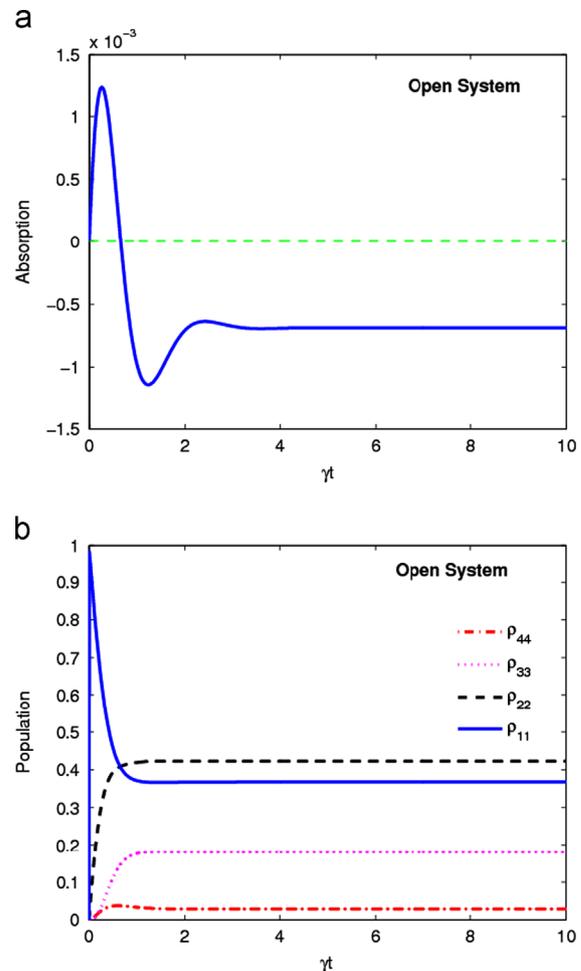


Fig. 8. Transient behaviors of (a) probe field absorption and (b) population distribution. The parameters values are $\Omega_c = 3\gamma$, $\Omega_s = \gamma$, $\Delta_p = \Delta_c = \Delta_s = 0$, $r_0 = 2\gamma$, $X = J_2/J_1 = 5$. The other parameters are the same as Fig. 2.

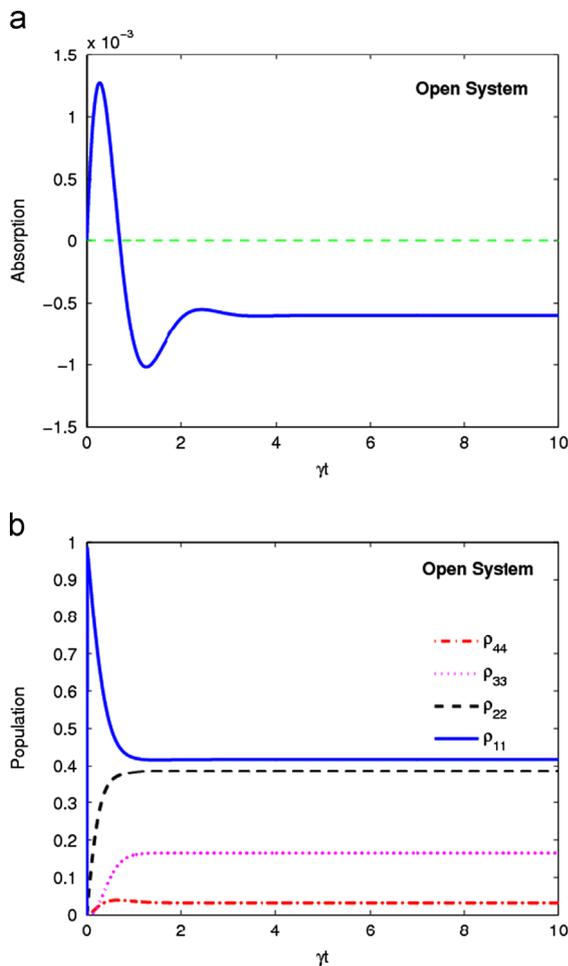


Fig. 9. Transient behaviors of (a) probe field absorption and (b) population distribution. The parameters values are $\Omega_c = 3\gamma$, $\Omega_s = \gamma$, $\Delta_p = \Delta_c = \Delta_s = 0$, $r_0 = 2\gamma$, $X = J_2/J_1 = 3$. The other parameters are the same as Fig. 2.

population distribution (Fig. 7(b)) are displayed. It is found that in this condition ($r_0 = J_1 = J_2 = 0$) the probe absorption increases for a short time, then steeply descend and eventually reaches a positive constant. In this case, most of the population remains in level |1). Therefore, the probe field experiences absorption, and population inversion doesn't appears. Now, we consider results for open system with parameters similar to Fig. 7 expect for $r_0 = 2$, $J_2 = 5J_1$. Hereon, the probe gain is obtained, that is to say the weak probe field will be amplified in open system (Fig. 8(a)). An investigation on Fig. 8(b) shows that the population distribution in level |1) reduces, while it increases in level |2). Accordingly, the population distribution in level |2) is more than level |1), i.e. $\rho_{22} > \rho_{11}$, thus population inversion occurs. It can be concluded that the gain is obtained in the presence of population inversion. In other words, lasing with inversion is achieved in open system via atomic exit rate. Now, we discuss the impact of ratio between atomic injection rates $X = J_2/J_1$ on the transient behavior of probe absorption and population distribution in Fig. 9 when the other parameters are chosen to be fixed as Fig. 8. It is obvious that for $X = J_2/J_1 = 3$, the probe absorption has a negative value (Fig. 9(a)), corresponds to probe gain. Furthermore, most of the population remains in level |1) (Fig. 9(b)). Thus, an inversionless gain is obtained. That is to say, lasing without population inversion is obtained in open four-level system via ratio of injection rates. Therefore, we showed that the open system changes from a state with population inversion to a state without population inversion.

This Flexibility and controllability of open system to achieve lasing with and without inversion shows its superiority than corresponding closed system which may provide some new possibilities for technological applications.

4. Conclusions

We have theoretically investigated transient response of the weak probe field in open four-level system and provide a comparison between its obtained results against corresponding closed system. We find that the transient evolution of the atomic response in each scheme shows different features. It is shown that in absence of atomic exit rate (closed system) the medium experiences absorption, while in presence of this parameter (open system) an amplification can be obtained. Moreover, we realized that atomic exit rate from cavity is an important characteristics of open system which can leads to high refractive index without absorption. Finally, we presented a novel method to achieve lasing with and without inversion only via cavity parameters i.e. atomic exit rate and the atomic injection rates, respectively. This way is completely different from what have been presented in closed atomic structures.

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