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Electromagnetically induced transparency and nonlinear pulse propagation in a combined tripod and Λ atom-light coupling scheme

To cite this article: H R Hamedi *et al* 2017 *J. Phys. B: At. Mol. Opt. Phys.* **50** 185401

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Electromagnetically induced transparency and nonlinear pulse propagation in a combined tripod and Λ atom-light coupling scheme

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Abstract
We consider propagation of a probe pulse in an atomic medium characterized by a combined tripod and Lambda (Λ) atom-light coupling scheme. The scheme involves three atomic ground states coupled to two excited states by five light fields. It is demonstrated that dark states can be formed for such an atom-light coupling. This is essential for formation of the electromagnetically induced transparency (EIT) and slow light. In the limiting cases the scheme reduces to conventional Λ- or N-type atom-light couplings providing the EIT or absorption, respectively. Thus, the atomic system can experience a transition from the EIT to the absorption by changing the amplitudes or phases of control lasers. Subsequently the scheme is employed to analyze the nonlinear pulse propagation using the coupled Maxwell–Bloch equations. It is shown that a generation of stable slow light optical solitons is possible in such a five-level combined tripod and Λ atomic system.

Keywords: electromagnetically induced transparency, dark state, combined tripod and Lambda atom-light coupling

(Some figures may appear in colour only in the online journal)

1. Introduction

Electromagnetically induced transparency (EIT) [1–6] plays an important role in controlling the propagation of light pulses in resonant media. Due to the EIT a weak probe beam of light tuned to an atomic resonance can propagate slowly and is almost lossless when the medium is driven by one or several control beams of light with a higher intensity [1–6]. The EIT is formed because the control and probe beams drive the atoms to their dark states representing a special superposition of the atomic ground states immune to the atom-light coupling. The absorption is suppressed due to a quantum mechanical interference between different excitation pathways of atomic energy levels leading to the EIT. The EIT has various important applications in quantum and nonlinear optics, such as slow and stored light [7–12], stationary light [13, 14], multiwave mixing [15–17], optical solitons [18–23], optical bistability [24, 25] and Kerr nonlinearity [26–30]. Using the slow light greatly enhances the light–matter interaction and enables nonlinear optical processes to achieve significant efficiency even at a single-photon level 26, 31–38].

There has been a considerable amount of activities on single- [2–4, 8, 11, 12, 39–43] and two-component (spinor) [6, 44–48] slow light in atomic media induced by the EIT. The former single-component slow light involves a probe beam of light and one or several control beams resonantly interacting with atomic media characterized by three level Lambda (Λ) type [2, 40] or four level tripod type [41, 42, 49–51] atom-light coupling schemes. In the later (spinor) case, double-tripod [6, 44–48] coupling schemes have been considered to support a simultaneous propagation of two probe beams leading to formation of a two-component slow light.
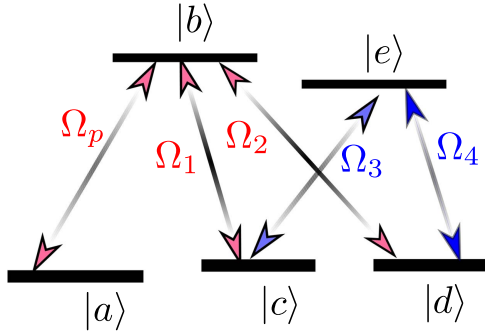
Figure 1. Schematic diagram of the five-level Lambda-tripod quantum system.

In this paper, we propose and analyze a novel five-level closed-loop scheme supporting the EIT. Closed-loop quantum configurations [30, 45, 52–55] represent a class of atom-light coupling schemes in which the driving fields acting on atoms build closed paths for the transitions between atomic levels. The interference between different paths makes the system sensitive to relative phases of the applied fields. In our proposal illustrated in figure 1, the atom-light coupling represents a five-level combined Lambda-tripod scheme, in which three atomic ground states are coupled to two excited states by four control and one probe laser fields. In other words, the scheme involves four atomic levels coupled between each other by four control fields and interacting with a ground level through a weak probe field. The existence of dark states, essential for the EIT, is analytically demonstrated for such an atom-light coupling setup. It is shown that in some specific limiting cases this scheme can be equivalent to the conventional Λ- or N-type atom-light couplings. An advantage of such an atomic system is a possibility of transitions to the Λ or N-type level schemes just by changing the amplitudes and phases of the control lasers. The limiting cases are discussed where the scheme reduces to atom-light couplings of the Λ- or N-type. By making a transition between the two limiting cases, one can switch from the EIT regime to the absorption for the probe field propagation.

Laser-driven atomic media, on the other hand, can be exploited to exhibit various nonlinear optical properties [15, 16, 18, 20, 24, 26, 28, 31, 56–58]. A particular example is formation of optical solitons with applications for optical buffers, phase shifters [59], switches [60], routers, transmission lines [61], wavelength converters [62], optical gates [63] and others. Solitons represent a specific type of stable shape-preserving waves propagating through nonlinear media. They can be formed due to a balance between dispersive and nonlinear effects leading to an undistorted propagation over long distance [63–73]. Following a report of ultraslow optical solitons in a highly resonant atomic medium by Wu and Deng [67], these solitary waves have received a considerable attention [19, 21–23, 74–82]. Here, the equations of motion that govern the nonlinear evolution of the probe pulse envelope are derived for the Lambda-tripod atom-light

coupling by solving the coupled Maxwell–Bloch equations. It is found that, by properly choosing the parameters of the system, the formation and slow propagation of shape-preserving optical solitons is feasible.

The paper is organized as follows. In section 2 we present the proposed setup and investigate propagation of the probe beam for various configurations of the control fields. Nonlinear propagation of the probe pulse is considered in section 3. Section 4 summarizes our findings.

2. Formulation and theoretical background

2.1. The system

Let us consider a probe pulse described by a Rabi frequency Ω_p. Additional laser fields described by Rabi frequencies Ω_1, Ω_2, Ω_3 and Ω_4 control propagation of the probe pulse. The probe and the control fields are assumed to co-propagate along the z direction. We shall analyze the light–matter interaction in an ensemble of atoms using a five-level Lambda-tripod scheme shown in figure 1. The atoms are characterized by three ground levels $|a\rangle$, $|c\rangle$ and $|d\rangle$, as well as two excited states $|b\rangle$ and $|e\rangle$. Four coherent control fields with the Rabi frequencies Ω_1, Ω_2, Ω_3 and Ω_4 induce dipole-allowed transitions $|b\rangle \longleftrightarrow |c\rangle$, $|b\rangle \longleftrightarrow |d\rangle$, $|e\rangle \longleftrightarrow |c\rangle$, and $|e\rangle \longleftrightarrow |d\rangle$, respectively. As a result, the control fields couple two excited states $|b\rangle$ and $|e\rangle$ via two different pathways $|b\rangle \overset{\Omega_1^*}{\rightarrow} |c\rangle \overset{\Omega_3}{\rightarrow} |e\rangle$ and $|b\rangle \overset{\Omega_2^*}{\rightarrow} |d\rangle \overset{\Omega_4}{\rightarrow} |e\rangle$ making a four level closed-loop coherent coupling scheme described by the Hamiltonian ($\hbar = 1$)

$$
\begin{aligned}
H_{\text{4Levels}} \\
= -\Omega_1^*|c\rangle\langle b| - \Omega_2^*|d\rangle\langle b| - \Omega_3^*|c\rangle\langle e| - \Omega_4^*|d\rangle\langle e| + \text{h.c.}
\end{aligned}
\tag{1}
$$

Furthermore, the tunable probe field with the Rabi frequency Ω_p induces a dipole-allowed optical transition $|a\rangle \longleftrightarrow |b\rangle$. The total Hamiltonian of the system involving all five atomic levels of the combined Λ and tripod level scheme is given by

$$
H_{\text{5Levels}} = -(\Omega_p^*|a\rangle\langle b| + \Omega_p|b\rangle\langle a|) + H_{\text{4Levels}}.
\tag{2}
$$

Note that the complex Rabi frequencies of the four control fields can be written as $\Omega_j = |\Omega_j|e^{i\phi_i}$, with $j = 1, 2, 3, 4$, where $|\Omega_j|$ and ϕ_j are the amplitude and phase of each applied field. As it will be explored below, in this scheme the destructive interference between different transition pathways induced by the control and probe beams can make the medium transparent for the resonant probe beams in a narrow frequency range due to the EIT. We define $\phi = (\phi_1 - \phi_2) - (\phi_3 - \phi_4)$ to be a relative phase among the four control fields forming a closed-loop coherent coupling. By changing ϕ, one can substantially modify the transparency and absorption properties for the probe field in such a Lambda-tripod scheme.

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2.2. Equations of motion

The dynamics of the probe field propagating through the atomic medium is described by the Maxwell–Bloch equations. To the first-order of Ω_p the equations have the form

$$\dot{\rho}_{ba}^{(1)} = d_1 \rho_{ba}^{(1)} + i\Omega_1 \rho_{ca}^{(1)} + i\Omega_2 \rho_{da}^{(1)} + i\Omega_p, \tag{3}$$

$$\dot{\rho}_{ca}^{(1)} = d_2 \rho_{ca}^{(1)} + i\Omega_1^* \rho_{ba}^{(1)} + i\Omega_3^* \rho_{ea}^{(1)}, \tag{4}$$

$$\dot{\rho}_{da}^{(1)} = d_2 \rho_{da}^{(1)} + i\Omega_2^* \rho_{ba}^{(1)} + i\Omega_4^* \rho_{ea}^{(1)}, \tag{5}$$

$$\dot{\rho}_{ea}^{(1)} = d_3 \rho_{ea}^{(1)} + i\Omega_3 \rho_{ca}^{(1)} + i\Omega_4 \rho_{da}^{(1)}, \tag{6}$$

and

$$\frac{\partial \Omega_p}{\partial z} + c^{-1} \frac{\partial \Omega_p}{\partial t} = i\eta \rho_{ba}^{(1)}, \quad \text{with} \quad \eta = \frac{2N\omega_p |\mu_{ba}|^2}{\hbar c}, \tag{7}$$

where $\rho_{u,v}^{(1)}$ are the first-order matrix elements of the density matrix operator $\rho = \sum |u\rangle \rho_{uv} \langle v|$. The optical Bloch equations (3)–(6) imply the probe field to be much weaker than the control ones. In that case most atomic population is in the ground state $|a\rangle$, and one can treat the probe field as a perturbation. Therefore, we can apply the perturbation expansion $\rho_{ij} = \Sigma_k \rho_{ij}^{(k)}$, where $\rho_{ij}^{(k)}$ represents the kth order part of ρ_{ij} in terms of probe field Ω_p. Since $\Omega_p \ll \Omega_i$ ($i = 1, 2, 3, 4$), the zeroth-order solution is $\rho_{aa}^{(0)} = 1$, while other elements being zero ($\rho_{bb}^{(0)} = \rho_{cc}^{(0)} = \rho_{dd}^{(0)} = \rho_{ee}^{(0)} = 0$). All fast-oscillating exponential factors associated with central frequencies and wave vectors have been eliminated from the equations, and only the slowly varying amplitudes are retained.

The wave equation (7) describes propagation of the probe field Ω_p influenced by the atomic medium, where μ_{ba} is an electric dipole matrix element corresponding to the transition $|b\rangle \longleftrightarrow |a\rangle$, N is the atomic density and ω_p is the frequency of the probe field. The density matrix equations (3)–(6) describe the evolution of the atomic system affected by the control and probe fields. They follow from the general quantum Liouville equation for the density matrix operator [83]

$$\dot{\rho} = -\frac{i}{\hbar}[H_{5\text{Levels}}, \rho] + L_\rho, \tag{8}$$

where the damping operator L_ρ describes the decay of the system described by parameters $d_1 = -\Gamma_b/2 + i\Delta_p$, $d_2 = i(\Delta_p - \Delta_2)$ and $d_3 = -\Gamma_e/2 + i(\Delta_p + \Delta_3 - \Delta_2)$ in equations (3)–(6). We have defined the detunings as: $\Delta_2 = \Delta_{bc} = \Delta_{bd}$, and $\Delta_3 = \Delta_{ec} = \Delta_{ed}$, with $\Delta_{bc} = \omega_1 - \omega_{bc}$, $\Delta_{bd} = \omega_2 - \omega_{bd}$, $\Delta_{ec} = \omega_3 - \omega_{ec}$, $\Delta_{ed} = \omega_4 - \omega_{ed}$, and $\Delta_p = \omega_p - \omega_{ba}$, where ω_i is a central frequency of the corresponding control field. Two excited states $|b\rangle$ and $|e\rangle$ decay with rates Γ_b and Γ_e, respectively.
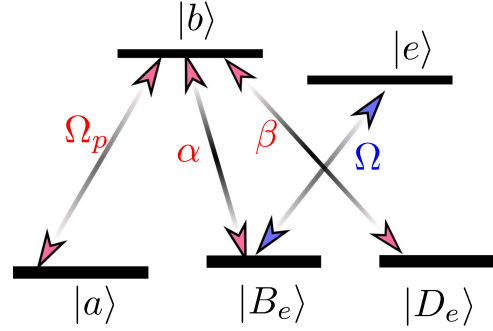

Figure 2. Five-level quantum system in the transformed basis for $\beta \neq 0$ and $\alpha \neq 0$.

2.3. Transition to a new basis

The Hamiltonian for the atomic four-level subsystem (1) can be represented as

$$H_{4\text{Levels}} = -\beta |D_e\rangle \langle b| - \alpha |B_e\rangle \langle b| - \Omega |B_e\rangle \langle e| + \text{h.c.}, \tag{9}$$

where

$$|D_e\rangle = \frac{1}{\Omega}(\Omega_4 |c\rangle - \Omega_3 |d\rangle), \tag{10}$$

$$|B_e\rangle = \frac{1}{\Omega}(\Omega_3^* |c\rangle + \Omega_4^* |d\rangle) \tag{11}$$

are the internal dark and bright states for the Λ- scheme made of the two ground states states $|c\rangle$ and $|d\rangle$, as well as an excited states $|e\rangle$. One can also introduce another set of dark and bright states corresponding to the Λ-scheme made of the same pair of ground states states $|c\rangle$ and $|d\rangle$, yet a different excited state $|b\rangle$:

$$|D_b\rangle = \frac{1}{\Omega}(\Omega_2 |c\rangle - \Omega_1 |d\rangle), \tag{12}$$

$$|B_b\rangle = \frac{1}{\Omega}(\Omega_1^* |c\rangle + \Omega_2^* |d\rangle). \tag{13}$$

In writing equation (9), the coefficient

$$\beta = \langle D_b | B_e \rangle = \langle D_e | B_d \rangle = \frac{1}{\Omega}(\Omega_1^* \Omega_4^* - \Omega_2^* \Omega_3^*) \tag{14}$$

represents the quantum inteference between the four control fields playing the main role in tuning dispersion and absorption properties in the combined tripod and Λ scheme. In addition, we define

$$\alpha = \langle D_b | D_e \rangle = \langle B_e | B_d \rangle = \frac{1}{\Omega}(\Omega_1^* \Omega_3 + \Omega_2^* \Omega_4) \tag{15}$$

and the total Rabi frequency

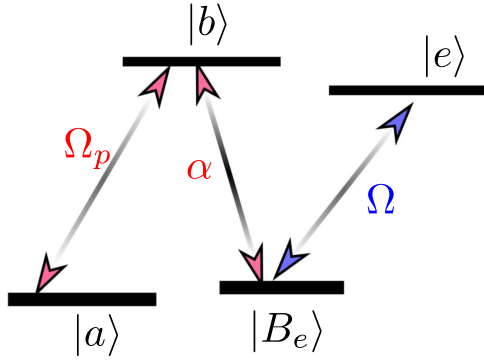$$\Omega = \sqrt{|\Omega_3|^2 + |\Omega_4|^2}. \tag{16}$$

Figure 3. The level scheme in the transformed basis for $\beta = 0$ and $\alpha \neq 0$. The ground state superposition $|D_e\rangle$ (now shown in the figure) is decoupled from the remaining four states.

Figure 4. Schematic diagram of the five-level quantum system for $\beta \neq 0$ and $\alpha = 0$.

By changing the quantum interference coefficient β and the coefficient α one arrives at three different situations. For each
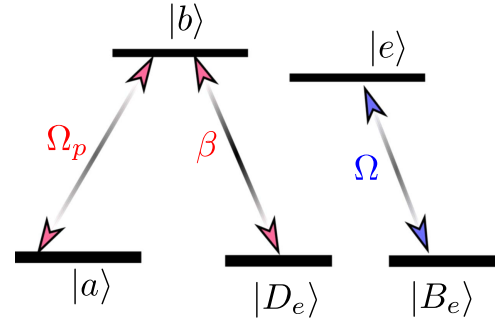
respectively. A steady-state solution to the density matrix element ρ_{ba} reads under the resonance condition $\Delta_2 = \Delta_3 = 0$

$$\rho_{ba}^{(1)} = \Omega_p \frac{\Delta_p\left(|\Omega_3|^2 + |\Omega_4|^2 + i\Delta_p\left(-\frac{\Gamma_e}{2} + i\Delta_p\right)\right)}{|\beta|^2\Omega^2 + i\Delta_p\left(-\frac{\Gamma_e}{2} + i\Delta_p\right)(|\Omega_1|^2 + |\Omega_2|^2) + i\Delta_p\left(-\frac{\Gamma_b}{2} + i\Delta_p\right)\left(|\Omega_3|^2 + |\Omega_4|^2 + i\Delta_p\left(-\frac{\Gamma_e}{2} + i\Delta_p\right)\right)},\tag{18}$$

of them we shall plot the level schemes in the basis involving the transformed states $|B_e\rangle$ and $|D_e\rangle$.

2.3.1. Situation (a): $\beta \neq 0$ and $\alpha \neq 0$. In the case when both coefficients α and β are nonzero, the five-level tripod and Λ scheme shown in figure 2 looks similar to the original scheme (figure 1) in the transformed basis, but the coupling between the states $|D_e\rangle$ and $|e\rangle$ is missing. The coefficient β is nonzero when $\Omega_1\Omega_4 \neq \Omega_2\Omega_3$. The condition $\beta \neq 0$ is valid provided $|\Omega_1||\Omega_4| \neq |\Omega_2||\Omega_3|$ and phase ϕ is arbitrary, or $|\Omega_1||\Omega_4| = |\Omega_2||\Omega_3|$ with $\phi \neq 0$.

When both α and β are nonzero, one can define a global dark state $|D\rangle$ for the whole atom-light coupling scheme

$$|D\rangle = \beta|a\rangle - \Omega_p|D_e\rangle.\tag{17}$$

The dark state is an eigenstate of the full atom-light Hamiltonian with a zero eigen-energy: $H_{5\text{Levels}}|D\rangle = 0$. The state $|D\rangle$ has no contribution by the ground state superposition $|B_e\rangle$, as well as no contribution by the bare excited states $|e\rangle$ and $|b\rangle$. As a result, there is no transition from the state $|D\rangle$ to the excited states $|e\rangle$ and $|b\rangle$, making the five-level closed-loop scheme transparent to the electromagnetic field. This is a new mechanism for the EIT compared with the Λ [2, 8], tripod [41, 42], or double tripod schemes [45, 46].

It is known that the real and imaginary parts of $\rho_{ba}^{(1)}$ correspond to the probe dispersion and absorption,

where the interference term β is involved.

A denominator of equation (18) represents the fourth order polynomial which contains zero points at four different detunings Δ_p of the probe field from the EIT resonance. This provides four maxima in the absorption profile of the system, as one can see in figure 5(a). Furthermore the EIT window is formed for zero detuning. In appendix A we have presented eigenstates and the corresponding eigenvalues of the Hamiltonian (9) describing the four-level subsystem. One can see that all four eigenstates $|n_i\rangle$ characterized by the eigenvalues $\lambda_{1,2} = \mp\frac{\sqrt{S-Y}}{\sqrt{2}}$ and $\lambda_{3,4} = \mp\frac{\sqrt{S+Y}}{\sqrt{2}}$ contain contributions due to the excited state $|b\rangle$ (note that S and Y can be found in appendix A). This results in four peaks in absorption profile of the system.

2.3.2. Situation (b): $\beta = 0$ and $\alpha \neq 0$. The condition $\beta = 0$ is fulfilled if $\Omega_1\Omega_4 = \Omega_2\Omega_3$, or equivalently $|\Omega_2||\Omega_3| = |\Omega_1||\Omega_4|$ and $\phi = 0$. In that case the state $|D_e\rangle$ is not involved, so the interaction Hamiltonian (9) for the four-level subsystem can be rewritten as

$$H_{4\text{Levels}} = -\alpha|B_e\rangle\langle b| - \Omega|B_e\rangle\langle e| + \text{h.c.}\tag{19}$$

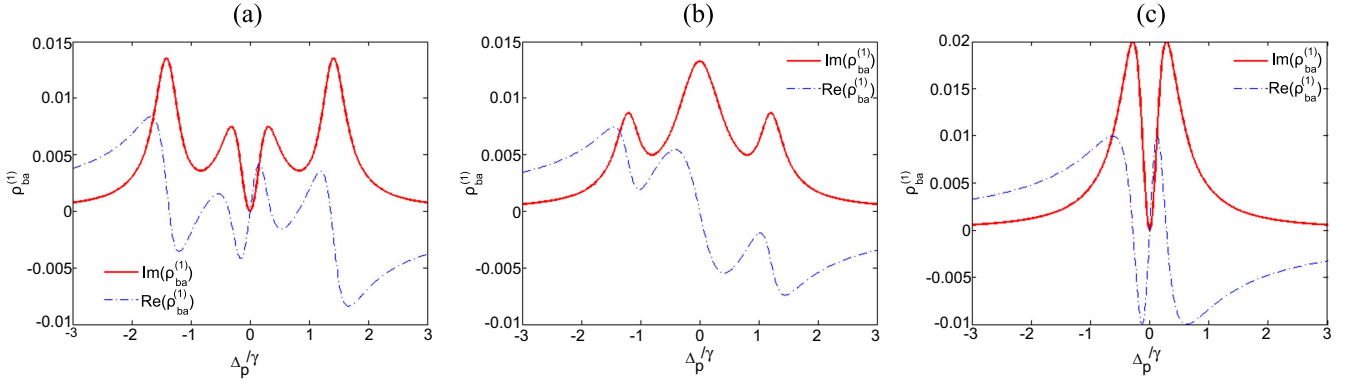Consequently, the five-level tripod and Λ scheme becomes

Figure 5. Probe absorption ($\text{Im}(\rho_{ba}^{(1)})$) versus Δ_p for (a) $\Omega_1 = 0.9\gamma$, $\Omega_2 = 0.7\gamma$, $\Omega_3 = 0.4\gamma$, $\Omega_4 = 0.8\gamma$, and $\phi = 0$ corresponding to the first situation, (b) $\Omega_1 = \Omega_2 = 0.5\gamma$, $\Omega_3 = \Omega_4 = 0.7\gamma$ and $\phi = 0$ corresponding to the second situation, and (c) $\Omega_1 = \Omega_2 = 0.2\gamma$, $\Omega_3 = \Omega_4 = 0.1\gamma$ and $\phi = \pi$ corresponding to the third situation. Other parameters are $\Gamma_e = \Gamma_b = \gamma$, $\Delta_2 = \Delta_3 = 0$, and $\Omega_p = 0.01\gamma$. Note that all frequencies are scaled by γ which should be of the order of MHz, like for cesium (Cs) atoms.

equivalent to a conventional *N*-type atomic system [29, 84] shown in figure 3.

Since the quantum interference term β vanishes, equation (18) simplifies to

$$\rho_{ba}^{(1)} = \Omega_p \frac{\left(|\Omega_3|^2 + |\Omega_4|^2 + i\Delta_p\left(-\frac{\Gamma_e}{2} + i\Delta_p\right)\right)}{i\left(-\frac{\Gamma_e}{2} + i\Delta_p\right)(|\Omega_1|^2 + |\Omega_2|^2) + i\left(-\frac{\Gamma_b}{2} + i\Delta_p\right)\left(|\Omega_3|^2 + |\Omega_4|^2 + i\Delta_p\left(-\frac{\Gamma_e}{2} + i\Delta_p\right)\right)}. \tag{20}$$

equation (18) for $\rho_{ba}^{(1)}$ then simplifies considerably, giving

$$\rho_{ba}^{(1)} = \Omega_p \frac{\Delta_p}{|\beta|^2 + i\Delta_p\left(-\frac{\Gamma_b}{2} + i\Delta_p\right)}. \tag{22}$$

The denominator of $\rho_{ba}^{(1)}$ in equation (20) is now a cubic polynomial, providing three absorption maxima. The eigenstates and eigenvalues corresponding to this situation are presented in appendix B. The eigenvector $|n_2\rangle$ characterized by a zero eigen-energy coincides with the dark state $|D_e\rangle$ and is decoupled from the radiation fields. Only the remaining three eigenvectors $|n_1\rangle$, $|n_3\rangle$ and $|n_4\rangle$ contain the contribution due to an excited state $|b\rangle$. This leads to three absorption peaks displayed in figure 5(b). In this way, the absence of quantum interference term β between the control fields destroys one of the peaks in the absorption profile leading to three absorption maxima.

2.3.3. Situation (c): $\beta \neq 0$ and $\alpha = 0$. When the coefficient β is nonzero but the coefficient α is zero, the interaction Hamiltonian (9) can be represented as

$$H_{4\text{Levels}} = -\beta|D_e\rangle\langle b| - \Omega|B_e\rangle\langle e| + \text{h.c.} \tag{21}$$

As illustrated in figure 4, the five-level tripod and Λ scheme is then equivalent to a conventional Λ-type atomic system [2, 40] which is decoupled from the two-level system involving the states $|B_e\rangle$ and $|e\rangle$.

In the following we consider a symmetric case where $|\Omega_1| = |\Omega_2|$, $|\Omega_3| = |\Omega_4|$ and $\phi = \pi$. In such a situation the conditions $\beta \neq 0$ and $\alpha = 0$ are fulfilled, with $\beta^2 = 2|\Omega_1^2|$.

Obviously, the polynominal in denominator becomes quadratic in Δ_p resulting in two absorption peaks or a single EIT window, which is a characteristic feature of the Λ scheme. Furthermore, the probe absorption and dispersion do not dependent on Ω, only β contributes to the optical properties. This is because the Λ-type scheme is now decoupled from the transition $|B_e\rangle \xrightarrow{\Omega} |e\rangle$, and the system behaves as a three-level Λ-type scheme containing $|a\rangle$, $|b\rangle$, and $|D_e\rangle$.

In the following we summarize our results for the behavior of real and imaginary parts of $\rho_{ba}^{(1)}$ corresponding to the probe dispersion and absorption for different situations (a)–(c) described above. Without a loss of generality, in all the simulations we take $\Gamma_e = \Gamma_b = \gamma$. Other frequencies are scaled by γ which should be in the order of MHz for cesium (Cs) atoms. Figure 5 shows that the our model provides a high control of dispersive-absorptive optical properties of the probe field. The absorption profile has four, three and two peaks featured in figures 5(a)–(c), respectively. There is the resulting change in the sign of the slope of the dispersion at $\Delta_p = 0$ for different situations (a)–(c). This gives rise to switching in the group velocity of the probe pulse from subluminal to superluminal or visa versa. In particular, for the choice of parameters satisfying situation (a) and (c), there is the subluminality accompanied by EIT at line center. On the other hand, the superluminality

accompanied by a considerable absorption is observed for the parametric condition satisfying the situation (b).

3. Linear and nonlinear pulse propagation in combined tripod and Λ scheme

In this section we consider propagation of the probe pulse in the proposed tripod and Λ scheme. Performing the time Fourier transform of equations (3)–(7) one can obtain

$$t_1(\omega)F_{ba} + \Omega_1 F_{ca} + \Omega_2 F_{da} + \Lambda_p = 0, \qquad (23)$$

$$t_2(\omega)F_{ca} + \Omega_1^* F_{ba} + \Omega_3^* F_{ea} = 0, \qquad (24)$$

$$t_2(\omega)F_{da} + \Omega_2^* F_{ba} + \Omega_4^* F_{ea} = 0, \qquad (25)$$

$$t_3(\omega)F_{ea} + \Omega_3 F_{ca} + \Omega_4 F_{da} = 0, \qquad (26)$$

and

$$\frac{\partial \Lambda_p}{\partial z} - \mathrm{i}\frac{\omega}{c}\Lambda_p = \mathrm{i}\eta F_{ba}, \qquad (27)$$

where $t_1(\omega) = \omega + \mathrm{i}\Gamma_b/2 + \Delta_p$, $t_2(\omega) = \omega + \Delta_p - \Delta_2$, and $t_3(\omega) = \omega + \mathrm{i}\Gamma_e/2 + \Delta_p - \Delta_2 + \Delta_3$. Note that F_{ij} and Λ_p represent the Fourier transforms of $\rho_{ij}^{(1)}$ and Ω_p, respectively, where ω is a deviation from the central frequency.

A solution of equation (27) is a plane wave of the form

$$\Lambda_p(z, \omega) = \Lambda_p(0, \omega)\mathrm{e}^{\mathrm{i}\kappa(\omega)z}, \qquad (28)$$

where

$$\kappa = \frac{\omega}{c} - \frac{\eta S_1(\omega)}{Q(\omega)}, \qquad (29)$$

describes the linear dispersion relation of the system. Expanding κ in power series around the center frequency of the probe pulse ($\omega = 0$) and taking only the first three terms, we get

$$\kappa = \kappa_0 + \kappa_1\omega + \kappa_2\omega^2, \qquad (30)$$

where the detailed expressions for the coefficients κ_0, κ_1 and κ_2 are given in appendix C, while $S_1(\omega)$ and $Q(\omega)$ can be found in appendix D. In equation (30), $\kappa_i = \frac{\mathrm{d}^j\kappa(\omega)}{\mathrm{d}\omega^j}|_{\omega=0}$ with $j = 0, 1, 2$ are the dispersion coefficients in different orders. In general, the real part of $\kappa_0 = \Upsilon + \mathrm{i}\chi/2$ defines the phase shift Υ per unit length, while the imaginary part indicates the linear absorption χ of the probe pulse. The group velocity v_g is given by $1/\kappa_1$, whereas the quadratic term κ_2 is associated with the group velocity dispersion which causes the pulse distortion.

In the linear regime, we take an incoming probe pulse to be of the Gaussian shape, $\Omega_p(0, t) = \Omega_p^0 \mathrm{e}^{-(t/\tau_0)^2}$, with a duration τ_0. The subsequent time evolution is obtained from equation (28) by carrying out an inverse Fourier transform [67]

$$\Omega_p(z, t) = \frac{\Omega_p^0}{\sqrt{L_z}} \exp\left[\mathrm{i}\kappa_0 z - \frac{(t - \kappa_1 z)^2}{L_z \tau_0^2}\right], \qquad (31)$$

with $L_z = s_1(z) - \mathrm{i}s_2(z)$, $s_1(z) = 1 + 4z\,\mathrm{Re}(\kappa_2)/\tau_0^2$ and $s_2(z) = 4z\,\mathrm{Im}(\kappa_2)/\tau_0^2$. In this way, even if there is no absorption due to EIT ($\mathrm{Im}(\kappa_i) = 0$, $i = 0, 1, 2$), the dispersion

effects can contribute to the pulse attenuation and spreading during propagation.

Our goal is to obtain shape-preserving optical pulses which can propagate without significant distortion and loss in our medium. The idea is to include the optical Kerr nonlinearity of the probe laser field into the light propagation, and show that the Kerr nonlinear effect can compensate the dispersion effects and result in shape-preserving optical pulses. To balance the dispersion effects and optical nonlinearity, in the following a theoretical model is employed based on the coupled Maxwell–Bloch equations for the nonlinear pulse propagation. Following [67], we take a trial function

$$\Lambda_p = \tilde{\Lambda}_p \mathrm{e}^{\mathrm{i}\kappa_0 z}. \qquad (32)$$

Substituting equation (32) into the wave equation (27) and using equations (29), (30) and (D.1) we obtain

$$\frac{\partial \Lambda_p}{\partial z}\mathrm{e}^{\mathrm{i}\kappa_0 z} = \mathrm{i}(\kappa_1\omega + \kappa_2\omega^2)\Lambda_p \mathrm{e}^{\mathrm{i}\kappa_0 z}, \qquad (33)$$

where we have replaced $\tilde{\Lambda}_p$ with Λ_p for the sake of convenience. Here we only keep terms up to the order ω^2 in expanding the dispersion relation κ_j.

In deriving the linearized wave equation (7), the nonlinear polarization due to the optical Kerr nonlinearity of the probe field has been neglected. Now we turn to investigate the nonlinear propagation of light due to the Kerr effect. To incorporate the nonlinear optical terms in the pulse propagation, the right-hand side of wave equation (7) must be rewritten as $\mathrm{i}\eta\rho_{ba}^{(1)} - \mathrm{i}\eta\mathrm{Kerr}$. The Kerr nonlinear term has an opposite sign than the linear term $\mathrm{i}\eta\rho_{ba}^{(1)}$ and the probe absorption and dispersion are proportional to imaginary and real parts of $\rho_{ba}^{(1)}$, respectively. Consequently, a large optical nonlinearity can cancel the dispersion and suppress the absorption of probe field, effectively. A derivation of the Kerr nonlinear coefficient is provided in appendix E.

Performing the inverse Fourier transform of equation (33), using the expression (E.11) for Kerr and introducing new coordinates $\zeta = z$, and $\eta = t - z/v_g$, we arrive at the nonlinear wave equation for the slowly varing envelope Ω_p

$$\mathrm{i}\frac{\partial}{\partial \zeta}\Omega_p - \kappa_2\frac{\partial^2}{\partial \eta^2}\Omega_p = \Theta\mathrm{e}^{-\chi\zeta}|\Omega_p|^2\Omega_p, \qquad (34)$$

where $\Theta = -\eta\mathrm{Kerr}$, and $\chi = 2\,\mathrm{Im}(\kappa_0) = 2\eta\,\mathrm{Im}\left(\frac{-S_1}{Q}\right)$.

Equation (34) contains generally complex coefficients. However, for suitable set of system parameters, the absorption coefficient χ may be very small, i.e., $\chi \simeq 0$, and imaginary parts of coefficients Θ and κ_2 may be made very small in comparison to their real parts, i.e., $\kappa_2 = \kappa_{2r} + \mathrm{i}\kappa_{2i} \approx \kappa_{2r}$, and $\Theta = \Theta_r + \mathrm{i}\Theta_i \approx \Theta_r$. In this case, equation (34) can be written as

$$\mathrm{i}\frac{\partial}{\partial \zeta}\Omega_p - \kappa_{2r}\frac{\partial^2}{\partial \eta^2}\Omega_p = \Theta_r|\Omega_p|^2\Omega_p. \qquad (35)$$

This represents the conventional nonlinear Schrödinger (NLS) equation which describes the nonlinear evolution of the probe pulse and allows bright and dark soliton solutions. The nature
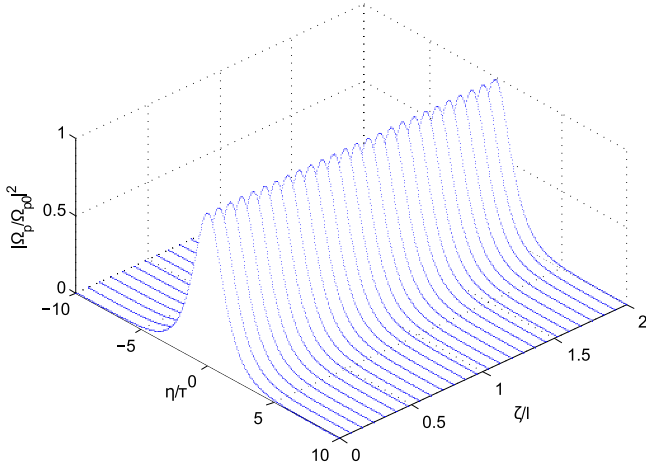
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Figure 6. Propagation dynamics of an ultraslow optical soliton with $\tau = 10^{-7}$ s, $l = 1$ cm, and $\eta = 1.0 \times 10^{10}$ cm^{-1} s^{-1} and the parameters given in the main text.

of the soliton solution is determined by a sign of the product $\kappa_{2r}\Theta_r$. A bright soliton is obtained for $\kappa_{2r}\Theta_r > 0$, and the solution is then given by

$$\Omega_p = \Omega_{p0}\,\text{sech}(\eta/\tau)\exp(-i\zeta\Theta_r|\Omega_p|^2/2). \tag{36}$$

For $\kappa_{2r}\Theta_r < 0$, one obtains the dark soliton solution of the form

$$\Omega_p = \Omega_{p0}\,\tanh(\eta/\tau)\exp(-i\zeta\Theta_r|\Omega_p|^2/2). \tag{37}$$

Here $\Omega_{p0} = (1/\tau)\sqrt{|\kappa_{2r}/\Theta_r|}$ represents an amplitude of the probe field and τ is the typical pulse duration (soliton width).

In the following, we explore a possibility for the formation of the shape preserving optical solitons in this combined tripod and Λ scheme for a realistic atomic system and present numeric calculations. The proposed scheme involving the five-level combined tripod and Λ structure can be experimentally implemented using the cesium (Cs) atom vapor. In our proposal, the levels $|c\rangle$, $|d\rangle$ and $|e\rangle$ can correspond to $|6S_{1/2}, F = 3, M_F = +1\rangle$, $|6S_{1/2}, F = 3, M_F = +3\rangle$ and $|6P_{3/2}, F = 2, M_F = +2\rangle$, respectively. In addition, the levels $|b\rangle$ and $|a\rangle$ can correspond to $|6P_{3/2}, F = 4\rangle$ and $|6S_{1/2}, F = 4\rangle$, respectively. The two excited states are assumed to decay with the rates $\Gamma_e = \Gamma_b = \gamma = 2\pi \times 5.2$ MHz.

Assuming the parameteric situation (a) described in the previous section, we take $|\Omega_1| = |\Omega_2| = 1.97 \times 10^9$ s^{-1}, $|\Omega_3| = 2.3 \times 10^9$ s^{-1}, $|\Omega_4| = 16.4 \times 10^7$ s^{-1}, $\Delta_2 = 6.4 \times 10^9$ s^{-1}, $\Delta_p = 5.9 \times 10^9$ s^{-1} and $\Delta_3 = 82 \times 10^7$ s^{-1}. Consequently, we obtain $\kappa_0 \approx (3.9 + 0.008i)$ cm^{-1}, $\kappa_1 \approx (4.7 - 2.1 \times 10^{-2}i)10^{-9}$ cm^{-1} s, $\kappa_2 \approx (-8.06 - 3.6 \times 10^{-2}i)10^{-17}$ cm^{-1} s^2, and $\Theta \approx (3.6 - 1.12 \times 10^{-2}i)10^{-19}$ cm^{-1} s^2. In this case, the standard NLS equation (35) with $\kappa_{2r}\Theta_r < 0$ is well characterized, leading to the formation of dark solitons in the proposed system. With this set of parameters, the fundamental soliton has a width and amplitude satisfying $|\Omega_{p0}\tau| = \sqrt{\kappa_{2r}/\Theta_r} \simeq 4.7$. As shown in figure 6, the dark soliton of this type remains fairly stable during propagation, which is due to the balance between the group-velocity dispersion and Kerr-type optical nonlinearity. According to equation (C.2) in appendix C, the group velocity v_g has a general

form $v_g^{-1} = c^{-1} + \eta(-g_1/Q + S_1 g_2/Q^2)$, with all coefficients given in appendix C. With the above system parameters, one can find $v_g \approx 7 \times 10^{-3}c$, indicating that the soliton propagates with a slow velocity.

The formation and propagation of such a slow light optical soliton in the system is due to the EIT condition described in the situation (a) of the previous section, i.e., $\beta \neq 0$ and $\alpha \neq 0$, or equivalently $\Omega_1\Omega_4 \neq \Omega_2\Omega_3$. Due to the EIT, the absorption of the probe field becomes negligibly small. In this case, an enhanced Kerr nonlinearity can compensate the dispersion effects in such a highly resonant medium resulting in shape preserving slow light optical solitons.

4. Concluding remarks

In conclusion we have demonstrated the existence of dark states which are essential for appearance of EIT for a situation where the atom-light interaction represents a five-level combined tripod and Λ configuration. The EIT is possible in the combined tripod and Λ scheme when the Rabi frequencies of the control fields obey the condition $\beta \neq 0$, $\alpha \neq 0$, where β and α given by equations (14) and (15) characterize the relative amplitudes and phases of the four control fields. Under this condition, the medium supports the lossless propagation of slow light. It is analytically demonstrated that combined tripod and Λ scheme can reduce to simpler atom-light coupling configurations under various quantum interference situations. In particular, this scheme is equivalent to a four-level N-type scheme when $\beta = 0$ and $\alpha \neq 0$. On the other hand, for $\beta \neq 0$ but $\alpha = 0$, a three level Λ-type atom-light coupling scheme can be established. As a result, by changing the Rabi frequencies of control fields, it is possible to make a transition from one limiting case to the another one. This can lead to switching from subluminality accompanied by EIT to superluminality along with absorption and visa versa. Based on the coupled Maxwell–Bloch equations, a nonlinear equation governing the evolution of the probe pulse envelope is then obtained. This leads to formation of stable optical solitons with a slow propagating velocity due to the balance between dispersion and Kerr nonlinearity of the system.

A possible realistic experimental realization of the proposed combined tripod and Λ setup can be implemented for the Cs atoms. The lower levels $|a\rangle$, $|c\rangle$ and $|d\rangle$ can then be assigned to $|6S_{1/2}, F = 4\rangle$, $|6S_{1/2}, F = 3, M_F = +1\rangle$ and $|6S_{1/2}, F = 3, M_F = +3\rangle$, respectively. Two excited states $|b\rangle$ and $|e\rangle$ can be attributed to the Cs states $|6P_{3/2}, F = 4\rangle$ and $|6P_{3/2}, F = 2, M_F = +2\rangle$, respectively.

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Appendix A. Eigenstates and eigenvalues for situation (a)

The expressions for the eigenstates and their corresponding eigenvalues for situation (a) are:

$$|n_1\rangle = \frac{\alpha^*(S-Y)}{\Omega(X-Y)}|b\rangle - \frac{\sqrt{S-Y}}{\sqrt{2}\,\Omega}|B_e\rangle$$
$$- \frac{\alpha^*\beta\sqrt{2(S-Y)}}{\Omega(X-Y)}|D_e\rangle + |e\rangle, \tag{A.1}$$

$$|n_2\rangle = \frac{\alpha^*(S-Y)}{\Omega(X-Y)}|b\rangle + \frac{\sqrt{S-Y}}{\sqrt{2}\,\Omega}|B_e\rangle$$
$$+ \frac{\alpha^*\beta\sqrt{2(S-Y)}}{\Omega(X-Y)}|D_e\rangle + |e\rangle, \tag{A.2}$$

$$|n_3\rangle = \frac{\alpha^*(S+Y)}{\Omega(X+Y)}|b\rangle - \frac{\sqrt{S+Y}}{\sqrt{2}\,\Omega}|B_e\rangle$$
$$- \frac{\alpha^*\beta\sqrt{2(S+Y)}}{\Omega(X+Y)}|D_e\rangle + |e\rangle, \tag{A.3}$$

$$|n_4\rangle = \frac{\alpha^*(S+Y)}{\Omega(X+Y)}|b\rangle + \frac{\sqrt{S+Y}}{\sqrt{2}\,\Omega}|B_e\rangle$$
$$+ \frac{\alpha^*\beta\sqrt{2(S+Y)}}{\Omega(X+Y)}|D_e\rangle + |e\rangle, \tag{A.4}$$

with eigenvalues

$$\lambda_1 = -\frac{\sqrt{S-Y}}{\sqrt{2}}, \tag{A.5}$$

$$\lambda_2 = \frac{\sqrt{S-Y}}{\sqrt{2}}, \tag{A.6}$$

$$\lambda_3 = -\frac{\sqrt{S+Y}}{\sqrt{2}}, \tag{A.7}$$

$$\lambda_4 = \frac{\sqrt{S+Y}}{\sqrt{2}}, \tag{A.8}$$

where

$$S = |\alpha|^2 + |\beta|^2 + \Omega^2, \tag{A.9}$$
$$X = |\alpha|^2 - |\beta|^2 + \Omega^2, \tag{A.10}$$
$$Y = \sqrt{S^2 - 4|\beta|^2\Omega^2}. \tag{A.11}$$

Appendix B. Eigenstates and eigenvalues for situation (b)

The expressions for the eigenstates and their corresponding eigenvalues for situation (b) are:

$$|n_1\rangle = -\frac{\Omega}{\alpha}|b\rangle + |e\rangle, \tag{B.1}$$

$$|n_2\rangle = |D_e\rangle, \tag{B.2}$$

$$|n_3\rangle = \frac{\alpha^*}{\Omega}|b\rangle - \frac{\sqrt{|\alpha|^2 + \Omega^2}}{\Omega}|B_e\rangle + |e\rangle, \tag{B.3}$$

$$|n_4\rangle = \frac{\alpha^*}{\Omega}|b\rangle + \frac{\sqrt{|\alpha|^2 + \Omega^2}}{\Omega}|B_e\rangle + |e\rangle, \tag{B.4}$$

with eigenvalues

$$\lambda_1 = 0, \tag{B.5}$$
$$\lambda_2 = 0, \tag{B.6}$$
$$\lambda_3 = -\sqrt{|\alpha|^2 + \Omega^2}, \tag{B.7}$$
$$\lambda_4 = \sqrt{|\alpha|^2 + \Omega^2}. \tag{B.8}$$

Appendix C. Explicit expressions for κ_0, $1/v_g$ and κ_2

Expressions for κ_0, $1/v_g$ and κ_2 read

$$\kappa_0 = \eta\frac{S_1}{Q}, \tag{C.1}$$

$$\frac{1}{v_g} = \frac{1}{c} + \eta\left(-\frac{g_1}{Q} + \frac{S_1 g_2}{Q^2}\right), \tag{C.2}$$

$$\kappa_2 = \eta\left(-\frac{g_3}{Q} + \frac{1}{Q^2}(2g_1 g_2 - S_1 g_4) - \frac{S_1 g_2^2}{Q^3}\right), \tag{C.3}$$

with

$$g_1 = t_2^2 + 2t_2 t_3 - (|\Omega_3|^2 + |\Omega_4|^2), \tag{C.4}$$

$$g_2 = (t_2 + t_3)(|\Omega_1|^2 + |\Omega_2|^2)$$
$$+ (t_1 + t_2)(|\Omega_3|^2 + |\Omega_4|^2) - t_1 t_2^2 - t_3 t_2^2 - 2t_1 t_2 t_3, \tag{C.5}$$

$$g_3 = -6\Delta_2 + 2\Delta_3 + 6\Delta_p + i\Gamma_e, \tag{C.6}$$

$$g_4 = -2(|\Omega_1|^2 + |\Omega_2|^2 + |\Omega_3|^2 + |\Omega_4|^2)$$
$$+ 2(2t_1 t_2 + t_1 t_3 + t_2 t_3) + 2t_2^2, \tag{C.7}$$

where t_1, t_2, t_3, S_1 and Q can be obtained by substituting $\omega = 0$ in coefficients $t_1(\omega)$, $t_2(\omega)$, $t_3(\omega)$, $S_1(\omega)$ and $Q(\omega)$, respectively.

Appendix D. Explicit expressions of F_{ba}, F_{ca}, F_{da} and F_{ea}

After some algebraic calculations, the solutions of equations (23)–(26) can be obtained as

$$F_{ba} = \frac{-\Lambda_p S_1(\omega)}{Q(\omega)}, \tag{D.1}$$

$$F_{ca} = \frac{\Lambda_p S_2(\omega)}{Q(\omega)}, \tag{D.2}$$

$$F_{da} = \frac{\Lambda_p S_3(\omega)}{Q(\omega)}, \tag{D.3}$$

$$F_{ea} = \frac{\Lambda_p S_4(\omega)}{Q(\omega)}, \tag{D.4}$$

where

$$S_1(\omega) = t_2^2(\omega)t_3(\omega) - t_2(\omega)(|\Omega_3|^2 + |\Omega_4|^2), \quad (D.5)$$

$$S_2(\omega) = |\Omega_1||\Omega_4|^2 - |\Omega_2||\Omega_3||\Omega_4|e^{-i\phi} - |\Omega_1|t_2(\omega)t_3(\omega), \quad (D.6)$$

$$S_3(\omega) = |\Omega_2||\Omega_3|^2 - |\Omega_1||\Omega_3||\Omega_4|e^{i\phi} - |\Omega_2|t_2(\omega)t_3(\omega), \quad (D.7)$$

$$S_4(\omega) = t_2(\omega)(|\Omega_2||\Omega_4| + |\Omega_1||\Omega_3|e^{i\phi}), \quad (D.8)$$

$$
\begin{aligned}
Q(\omega) = {} & t_2(\omega)t_3(\omega)(|\Omega_1|^2 + |\Omega_2|^2) \\
& + t_1(\omega)t_2(\omega)(|\Omega_3|^2 + |\Omega_4|^2) \\
& - t_1(\omega)t_2^2(\omega)t_3(\omega) - |\Omega_2|^2|\Omega_3|^2 - |\Omega_1|^2|\Omega_4|^2 \\
& + 2|\Omega_1||\Omega_2||\Omega_3||\Omega_4|\cos(\phi).
\end{aligned}
\quad (D.9)
$$

Appendix E. Kerr nonlinear coefficient

One may write the Maxwell equations under the slowly varying envelope approximation as

$$\frac{\partial \Omega_p}{\partial z} + c^{-1}\frac{\partial \Omega_p}{\partial t} = i\eta \Phi_b \Phi_a^*, \quad (E.1)$$

where $\Phi_b \Phi_a^* = \rho_{ba}$, as well as Φ_a and Φ_b (together with Φ_c, Φ_d and Φ_e) represent the amplitudes of atomic wavefunctions for each atomic state and satisfy the relation

$$|\Phi_a|^2 + |\Phi_b|^2 + |\Phi_c|^2 + |\Phi_d|^2 + |\Phi_e|^2 = 1. \quad (E.2)$$

Initially all atoms are assumed to be in the ground state $|a\rangle$. As the Rabi frequency of the probe field is much weaker than that of the control fields, one can neglect the depletion of ground level $|a\rangle$, and one has $\Phi_a \approx 1$. Adopting a perturbation treatment of the system response to the first order of probe field, we can take $\Phi_L = \Sigma_L \Phi_L^{(k)}$ ($L = a, b, c, d, e$). Here $\Phi_L^{(k)}$ is the kth order part of Φ_L in terms of Ω_p, where $\Phi_b^{(0)} = \Phi_c^{(0)} = \Phi_d^{(0)} = \Phi_e^{(0)} = 0$, and $\Phi_a^{(0)} = 1$, while $\Phi_a^{(1)} = 0$. Thus, to the first order in Ω_p we may write

$$\Phi_a = \Phi_a^{(0)}, \quad (E.3)$$

$$\Phi_b \Phi_a^* = \Phi_b^{(1)}\Phi_a^{(0)*} = \Phi_b^{(1)}\Phi_a^{(0)} = \Phi_b^{(1)} = \rho_{ba}^{(1)}, \quad (E.4)$$

$$\Phi_c \Phi_a^* = \Phi_c^{(1)}\Phi_a^{(0)*} = \Phi_c^{(1)}\Phi_a^{(0)} = \Phi_c^{(1)} = \rho_{ca}^{(1)}, \quad (E.5)$$

$$\Phi_d \Phi_a^* = \Phi_d^{(1)}\Phi_a^{(0)*} = \Phi_d^{(1)}\Phi_a^{(0)} = \Phi_d^{(1)} = \rho_{da}^{(1)}, \quad (E.6)$$

$$\Phi_e \Phi_a^* = \Phi_e^{(1)}\Phi_a^{(0)*} = \Phi_e^{(1)}\Phi_a^{(0)} = \Phi_e^{(1)} = \rho_{ea}^{(1)}. \quad (E.7)$$

In this limit equation (E.2) reduces to

$$|\Phi_a^{(0)}|^2 + |\Phi_b^{(1)}|^2 + |\Phi_c^{(1)}|^2 + |\Phi_d^{(1)}|^2 + |\Phi_e^{(1)}|^2 = 1. \quad (E.8)$$

Using equations (E.3)–(E.8), the right-hand side of wave equation (E.1) can be represented as

$$
\begin{aligned}
i\eta \Phi_b \Phi_a^* &= i\eta \Phi_b^{(1)}|\Phi_a^{(0)}|^2 \\
&= i\eta \Phi_b^{(1)}[1 - (|\Phi_b^{(1)}|^2 + |\Phi_c^{(1)}|^2 + |\Phi_d^{(1)}|^2 + |\Phi_e^{(1)}|^2)] \\
&= i\eta \rho_{ba}^{(1)} - i\eta \rho_{ba}^{(1)}(|\rho_{ba}^{(1)}|^2 + |\rho_{ca}^{(1)}|^2 + |\rho_{da}^{(1)}|^2 + |\rho_{ea}^{(1)}|^2).
\end{aligned}
\quad (E.9)
$$

The first term $i\eta \rho_{ba}^{(1)}$ shows the linear part of the right-hand side of wave equation (E.1) which was featured in equation (7). In addition, $-i\eta\text{Kerr} = -i\eta \rho_{ba}^{(1)}(|\rho_{ba}^{(1)}|^2 + |\rho_{ca}^{(1)}|^2 + |\rho_{da}^{(1)}|^2 + |\rho_{ea}^{(1)}|^2)$ represents the nonlinear part of the right-hand side of wave equation (E.1). From this expression, the explicit form of the nonlinear coefficient Kerr can be readily derived as

$$\text{Kerr} = \rho_{ba}^{(1)}(|\rho_{ba}^{(1)}|^2 + |\rho_{ca}^{(1)}|^2 + |\rho_{da}^{(1)}|^2 + |\rho_{ea}^{(1)}|^2). \quad (E.10)$$

As F_{ij} is the Fourier transform of $\rho_{ij}^{(1)}$, the coefficients for $\rho_{ij}^{(1)}$ can be obtained by taking $\omega = 0$ in the coefficients given in appendix D. Replacing the coefficients for $\rho_{ij}^{(1)}$ in this way into equation (E.10) yields

$$\text{Kerr} = \frac{-S_1}{Q|Q|^2}(|S_1|^2 + |S_2|^2 + |S_3|^2 + |S_4|^2). \quad (E.11)$$

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References

[1] Arimondo E 1996 *Progress in Optics* (Elsevier: Amsterdam)
[2] Harris S E 1997 *Phys. Today* **50** 36
[3] Lukin M D 2003 *Rev. Mod. Phys.* **75** 457–72
[4] Fleischhauer M, Imamoglu A and Marangos J P 2005 *Rev. Mod. Phys.* **77** 633–73
[5] Wu Y and Yang X 2005 *Phys. Rev. A* **71** 053806
[6] Fleischhauer M and Juzeliūnas G 2016 Slow, stored and stationary light *Optics in Our Time* ed M D Al-Amri *et al* (Cham: Springer) pp 359–83
[7] Hau L V, Harris S E, Dutton Z and Behroozi C H 1999 *Nature* **397** 594
[8] Fleischhauer M and Lukin M D 2000 *Phys. Rev. Lett.* **84** 5094–7
[9] Liu C, Dutton Z, Behroozi C H and Hau L V 2001 *Nature* **409** 490
[10] Phillips D F, Fleischhauer A, Mair A, Walsworth R L and Lukin M D 2001 *Phys. Rev. Lett.* **86** 783
[11] Fleischhauer M and Lukin M D 2002 *Phys. Rev. A* **65** 022314
[12] Juzeliūnas G and Carmichael H J 2002 *Phys. Rev. A* **65** 021601
[13] Bajcsy M, Zibrov A S and Lukin M D 2003 *Nature* **426** 638
[14] Lin Y W, Liao W T, Peters T, Chou H C, Wang J S, Cho H W, Kuan P C and Yu I A 2009 *Phys. Rev. Lett.* **102** 213601
[15] Wu Y, Saldana J and Zhu Y 2003 *Phys. Rev. A* **67** 013811
[16] Zhang Y, Anderson B and Xiao M 2008 *Phys. Rev. A* **77** 061801
[17] Zhang Y, Khadka U, Anderson B and Xiao M 2009 *Phys. Rev. Lett.* **102** 013601
[18] Wu Y and Deng L 2004 *Phys. Rev. Lett.* **93** 143904
[19] Huang G, Deng L and Payne M G 2005 *Phys. Rev. E* **72** 016617
[20] Li L and Huang G 2010 *Phys. Rev. A* **82** 023809
[21] Si L G, Yang W X, Lü X Y, Hao X and Yang X 2010 *Phys. Rev. A* **82** 013836
[22] Yang W X, Chen A X, Lee R K and Wu Y 2011 *Phys. Rev. A* **84** 013835
[23] Chen Y, Bai Z and Huang G 2014 *Phys. Rev. A* **89** 023835
[24] Joshi A, Brown A, Wang H and Xiao M 2003 *Phys. Rev. A* **67** 041801

[25] Li J H, Lü X Y, Luo J M and Huang Q J 2006 *Phys. Rev. A* **74** 035801
[26] Schmidt H and Imamoglu A 1996 *Opt. Lett.* **21** 1936–8
[27] Wang H, Goorskey D and Xiao M 2002 *Opt. Lett.* **27** 258–60
[28] Niu Y and Gong S 2006 *Phys. Rev. A* **73** 053811
[29] Sheng J, Yang X, Wu H and Xiao M 2011 *Phys. Rev. A* **84** 053820
[30] Hamedi H R and Juzeliūnas G 2015 *Phys. Rev. A* **91** 053823
[31] Harris S E and Yamamoto Y 1998 *Phys. Rev. Lett.* **81** 3611–4
[32] Lukin M D and Imamoğlu A 2000 *Phys. Rev. Lett.* **84** 1419–22
[33] Wang Z B, Marzlin K P and Sanders B C 2006 *Phys. Rev. Lett.* **97** 063901
[34] Shiau B W, Wu M C, Lin C C and Chen Y C 2011 *Phys. Rev. Lett.* **106** 193006
[35] Chen Y H, Lee M J, Hung W, Chen Y C, Chen Y F and Yu I A 2012 *Phys. Rev. Lett.* **108** 173603
[36] Vivek Venkataraman K S and Gaeta A L 2013 *Nat. Photon.* **7** 138–41
[37] Maxwell D, Szwer D J, Paredes-Barato D, Busche H, Pritchard J D, Gauguet A, Weatherill K J, Jones M P A and Adams C S 2013 *Phys. Rev. Lett.* **110** 103001
[38] Baur S, Tiarks D, Rempe G and Dürr S 2014 *Phys. Rev. Lett.* **112** 073901
[39] Boller K J, Imamoğlu A and Harris S E 1991 *Phys. Rev. Lett.* **66** 2593–6
[40] Ruseckas J, Juzeliūnas G, Öhberg P and Barnett S M 2007 *Phys. Rev. A* **76** 053822
[41] Ruseckas J, Mekys A and Juzeliūnas G 2011 *Phys. Rev. A* **83** 023812
[42] Paspalakis E and Knight P L 2002 *J. Opt. B: Quantum Semiclass. Opt.* **4** S372
[43] Schnorrberger U, Thompson J D, Trotzky S, Pugatch R, Davidson N, Kuhr S and Bloch I 2009 *Phys. Rev. Lett.* **103** 033003
[44] Unanyan R G, Otterbach J, Fleischhauer M, Ruseckas J, Kudriašov V and Juzeliūnas G 2010 *Phys. Rev. Lett.* **105** 173603
[45] Ruseckas J, Kudriašov V, Yu I A and Juzeliūnas G 2013 *Phys. Rev. A* **87** 053840
[46] Lee M J, Ruseckas J, Lee C Y, Kudriasov V, Chang K F, Cho H W, Juzeliūnas G and Yu I A 2014 *Nat. Commun.* **5** 5542
[47] Ruseckas J, Kudriašov V, Juzeliūnas G, Unanyan R G, Otterbach J and Fleischhauer M 2011 *Phys. Rev. A* **83** 063811
[48] Bao Q Q, Zhang X H, Gao J Y, Zhang Y, Cui C L and Wu J H 2011 *Phys. Rev. A* **84** 063812
[49] Raczynski A, Rzepecka M, Zaremba J and Zielinska-Kaniasty S 2006 *Opt. Commun.* **260** 73
[50] Raczynski A, Zaremba J and Zielinska-Kaniasty S 2007 *Phys. Rev. A* **75** 013810
[51] Beck S and Mazets I E 2017 *Phys. Rev. A* **95** 013818
[52] Payne M G and Deng L 2002 *Phys. Rev. A* **65** 063806
[53] Korsunsky E A and Kosachiov D V 1999 *Phys. Rev. A* **60** 4996–5009
[54] Shpaisman H, Wilson-Gordon A D and Friedmann H 2005 *Phys. Rev. A* **71** 043812
[55] Fleischhaker R and Evers J 2008 *Phys. Rev. A* **77** 043805
[56] Dey V R K, Evers T N and Kiffner M J 2015 *Phys. Rev. A* **92** 023840
[57] Braje D A, Balić V, Goda S, Yin G Y and Harris S E 2004 *Phys. Rev. Lett.* **93** 183601
[58] Dey T N and Agarwal G S 2007 *Phys. Rev. A* **76** 015802
[59] Kang H and Zhu Y 2003 *Phys. Rev. Lett.* **91** 093601
[60] Rodrigo A, Vicencio M I M and Kivshar Y S 2003 *Opt. Lett.* **28** 1942–4
[61] Heebner E, Boyd J and Park Q H R W 2002 *Phys. Rev. E* **65** 036619
[62] Andrea Melloni F M and Martinelli M 2003 *Opt. Photonics News* **14** 44–8
[63] Liu X J, Jing H and Ge M L 2004 *Phys. Rev. A* **70** 055802
[64] Agrawal G P 2001 *Nonlinear Fiber Optics* 3rd edn (New York: Academic)
[65] Hasegawa A and Matsumoto M 2003 *Optical Solitons in Fibers* (Berlin: Springer)
[66] Xie X T, Li W B and Yang W X 2006 *J. Phys. B: At. Mol. Opt. Phys.* **39** 401
[67] Wu Y and Deng L 2004 *Phys. Rev. Lett.* **93** 143904
[68] Burger S, Bongs K, Dettmer S, Ertmer W, Sengstock K, Sanpera A, Shlyapnikov G V and Lewenstein M 1999 *Phys. Rev. Lett.* **83** 5198–201
[69] Denschlag J *et al* 2000 *Science* **287** 97
[70] Huang G, Szeftel J and Zhu S 2002 *Phys. Rev. A* **65** 053605
[71] Kivshar Y S and Luther-Davies B 1998 *Phys. Rep.* **298** 81
[72] Lin Y and Lee R K 2007 *Opt. Express* **15** 8781
[73] Xie X T, Li W, Li J, Yang W X, Yuan A and Yang X 2007 *Phys. Rev. B* **75** 184423
[74] Yang W X, Hou J M, Lin Y and Lee R K 2009 *Phys. Rev. A* **79** 033825
[75] Wu Y and Deng L 2004 *Opt. Lett.* **29** 2064
[76] Hang C, Huang G and Deng L 2006 *Phys. Rev. E* **74** 046601
[77] Si L G, Yang W X, Liu J B, Li J and Yang X 2009 *Opt. Express* **17** 7771
[78] Hang C and Huang G 2010 *Opt. Express* **18** 2952
[79] Zhu C and Huang G 2011 *Opt. Express* **19** 1963
[80] Liu J B, Liu N, Shan C J, Liu T K and Huang Y X 2010 *Phys. Rev. E* **81** 036607
[81] Li L and Huang G 2010 *Phys. Rev. A* **82** 023809
[82] Chen Y, Chen Z and Huang G 2015 *Phys. Rev. A* **91** 023820
[83] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge University Press)
[84] Harris S E and Hau L V 1999 *Phys. Rev. Lett.* **82** 4611–4