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Influence of the finite duration of a measurement on the quantum Zeno effect

Julius Ruseckas

Institute of Theoretical Physics and Astronomy, A. Goštauto 12, 2600 Vilnius, Lithuania

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Abstract

We analyze the influence of the finite duration of the measurement on the quantum Zeno effect, using a simple model of the measurement. It is shown that the influence of the finite duration of the measurement is unimportant when this duration is small compared to the duration of the free evolution between the measurements. © 2001 Published by Elsevier Science B.V.

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1. Introduction

The quantum Zeno effect is a consequence of the influence of the measurements on the evolution of a quantum system. In quantum mechanics the short-time behavior of the non-decay probability of unstable particle is not exponential but quadratic [1]. This deviation from the exponential decay has been observed experimentally [2,3]. In 1977, Mishra and Sudarshan [4] showed that this behavior when combined with the quantum theory of measurement, based on the assumption of the collapse of the wave function, led to a very surprising conclusion: frequent observations slowed down the decay. They modeled the continuous observation of the system by a succession of the instantaneous measurements with free evolution of the system between the measurements.

Cook [5] suggested an experiment on the quantum Zeno effect that was realized by Itano et al. [6]. In this experiment a repeatedly measured two-level system undergoing Rabi oscillations has been used. The outcome of this experiment has also been explained without the collapse hypothesis [7–9]. Recently, an experiment similar to Ref. [6] has been performed by Balzer et al. [10]. The quantum Zeno and anti-Zeno effects have been experimentally observed in Ref. [3].

In the analysis of the quantum Zeno effect the finite duration of the measurement becomes important. In Ref. [11] a simple model that allows us to take into account the finite duration and finite accuracy of the measurement has been developed. However, in Ref. [11] it has been analyzed the case when there are no free evolution between the measurements. In this Letter we obtain the corrections to the jump probability due to the finite duration of the measurement with the free evolution between the measurements.

We proceed as follows. In Section 2 we present the model of the measurement. Section 3 is devoted to

E-mail address: ruseckas@itpa.lt (J. Ruseckas).

the derivation of the formula for the probability of the jump into another level during the measurement of the frequently measured perturbed system. In Section 4 the evolution of the measured two-level system is analysed as an example of the application of our model. Section 5 summarizes our findings.

2. Model of the measurements

We consider a system which consists of two parts. The first part of the system has the discrete energy spectrum. The Hamiltonian of this part is \widehat{H}_0 . The other part of the system is represented by Hamiltonian \widehat{H}_1 . Hamiltonian \widehat{H}_1 commutes with \widehat{H}_0 . In a particular case the second part may be absent and \widehat{H}_1 may be zero. The operator $\widehat{V}(t)$ causes the jumps between the different energy levels of \widehat{H}_0 . Therefore, the full Hamiltonian of the system equals to $\widehat{H}_S = \widehat{H}_0 + \widehat{H}_1 + \widehat{V}(t)$. An example of such a system is an atom with the Hamiltonian \widehat{H}_0 interacting with the electromagnetic field, represented by \widehat{H}_1 .

We will measure in which eigenstate of the Hamiltonian \widehat{H}_0 the system is. The measurement is performed by coupling the system with the detector. The full Hamiltonian of the system and the detector equals to

$$\widehat{H} = \widehat{H}_S + \widehat{H}_D + \widehat{H}_I, \quad (1)$$

where \widehat{H}_D is the Hamiltonian of the detector and \widehat{H}_I represents the interaction between the detector and the system. We choose the operator \widehat{H}_I in the form

$$\widehat{H}_I = \lambda \hat{q} \widehat{H}_0, \quad (2)$$

where \hat{q} is the operator acting in the Hilbert space of the detector and the parameter λ describes the strength of the interaction. This system–detector interaction is considered by von Neumann [12] and in Refs. [11, 13–18]. In order to obtain a sensible measurement, the parameter λ must be large. We require a continuous spectrum of operator \hat{q} . For simplicity, we can consider the quantity q as the coordinate of the detector.

The measurement begins at time moment t_0 . At the beginning of the interaction with the detector, the detector is in the pure state $|\Phi\rangle$. The full density matrix of the system and detector is $\hat{\rho}(t_0) = \hat{\rho}_S(t_0) \otimes |\Phi\rangle\langle\Phi|$ where $\hat{\rho}_S(t_0)$ is the density matrix of the system. The duration of the measurement

is τ . After the measurement the density matrix of the system is $\hat{\rho}_S(\tau + t_0) = \text{Tr}_D\{\widehat{U}_M(\tau, t_0)(\hat{\rho}_S(t_0) \otimes |\Phi\rangle\langle\Phi|)\widehat{U}_M^\dagger(\tau, t_0)\}$, where $\widehat{U}_M(t, t_0)$ is the evolution operator of the system and detector, obeying the equation

$$i\hbar \frac{\partial}{\partial t} \widehat{U}_M(t, t_0) = \widehat{H}(t + t_0) \widehat{U}_M(t, t_0) \quad (3)$$

with the initial condition $\widehat{U}_M(0, t_0) = 1$. Further, for simplicity we will neglect the Hamiltonian of the detector (as in Ref. [11]). Then the evolution operator \widehat{U}_M obeys the equation

$$i\hbar \frac{\partial}{\partial t} \widehat{U}_M(t, t_0) = ((1 + \lambda \hat{q}) \widehat{H}_0 + \widehat{H}_1 + \widehat{V}(t + t_0)) \widehat{U}_M(t, t_0). \quad (4)$$

After the measurement the system is left for the measurement-free evolution for time $T - \tau$. The density matrix becomes $\hat{\rho}_S(T + t_0) = \widehat{U}_F(T - \tau, \tau + t_0) \hat{\rho}_S(\tau + t_0) \widehat{U}_F^\dagger(T - \tau, \tau + t_0)$, where $\widehat{U}_F(t, t_0)$ is the evolution operator of the system only, obeying the equation

$$i\hbar \frac{\partial}{\partial t} \widehat{U}_F(t, t_0) = \widehat{H}_S(t + t_0) \widehat{U}_F(t, t_0) \quad (5)$$

with the initial condition $\widehat{U}_F(0, t_0) = 1$.

The measurements of the duration τ with a subsequent free evolution for the time $T - \tau$ are repeated many times with the measurement period T . Such a process was considered by the Mishra and Sudarshan [4] and realized in the experiments [6].

3. Jump probability

We will calculate the probability of the jump from the initial to the final state during the measurement and subsequent measurement-free evolution. The jumps are induced by the operator $\widehat{V}(t)$ that represents the perturbation of the unperturbed Hamiltonian $\widehat{H}_0 + \widehat{H}_1$. We will take into account the influence of the operator \widehat{V} by the perturbation method, assuming that the durations of the measurement τ and of the free evolution $T - \tau$ are small.

The operator $\widehat{V}(t)$ in the interaction picture during the measurement is

$$\widetilde{V}_M(t, t_0) = \widehat{U}_M^{(0)}(t) \widehat{V}(t + t_0) \widehat{U}_M^{(0)}(t), \quad (6)$$

where $\widehat{U}_M^{(0)}(t)$ is the evolution operator of the system and the detector (1) without the perturbation \widehat{V}

$$\widehat{U}_M^{(0)}(t) = \exp\left(-\frac{i}{\hbar}(\widehat{H}_0 + \widehat{H}_1 + \widehat{H}_1)t\right). \quad (7)$$

The evolution operator $\widehat{U}_M(\tau, t_0)$ in the second order approximation equals to

$$\begin{aligned} \widehat{U}_M(\tau, t_0) \approx & \widehat{U}_M^{(0)}(\tau) \left(1 + \frac{1}{i\hbar} \int_0^\tau dt \widetilde{V}_M(t, t_0) \right. \\ & \left. - \frac{1}{\hbar^2} \int_0^\tau dt_1 \int_0^{t_1} dt_2 \widetilde{V}_M(t_1, t_0) \widetilde{V}_M(t_2, t_0) \right). \end{aligned} \quad (8)$$

The operator $\widehat{V}(t)$ in the interaction picture during the free evolution is

$$\widetilde{V}_F(t, t_0) = \widehat{U}_F^{(0)}(t) \widehat{V}(t + t_0) \widehat{U}_F^{(0)}(t), \quad (9)$$

where $\widehat{U}_F^{(0)}(t)$ is the evolution operator of the system without the perturbation \widehat{V} , i.e.,

$$\widehat{U}_F^{(0)}(t) = \exp\left(-\frac{i}{\hbar}(\widehat{H}_0 + \widehat{H}_1)t\right). \quad (10)$$

The evolution operator $\widehat{U}_F(t, t_0)$ in the second order approximation equals to

$$\begin{aligned} \widehat{U}_F(t, t_0) \approx & \widehat{U}_F^{(0)}(t) \left(1 + \frac{1}{i\hbar} \int_0^t dt_1 \widetilde{V}_F(t_1, t_0) \right. \\ & \left. - \frac{1}{\hbar^2} \int_0^t dt_1 \int_0^{t_1} dt_2 \widetilde{V}_F(t_1, t_0) \widetilde{V}_F(t_2, t_0) \right). \end{aligned} \quad (11)$$

We can choose the basis $|n\alpha\rangle$ common for the operators \widehat{H}_0 and \widehat{H}_1 ,

$$\widehat{H}_0|n\alpha\rangle = E_n|n\alpha\rangle, \quad (12)$$

$$\widehat{H}_1|n\alpha\rangle = E_1(n, \alpha)|n\alpha\rangle, \quad (13)$$

where n numbers are the eigenvalues of Hamiltonian \widehat{H}_0 and α represents the remaining quantum numbers.

The probability of the jump from the level $|i\alpha\rangle$ to the level $|f\alpha_1\rangle$ is

$$\begin{aligned} W(i\alpha \rightarrow f\alpha_1) & = \text{Tr}_D \{ \langle f\alpha_1 | \widehat{U}_F(T - \tau) \widehat{U}_M(\tau) (|i\alpha\rangle\langle i\alpha| \otimes |\Phi\rangle\langle\Phi|) \\ & \quad \times \widehat{U}_F^\dagger(T - \tau) \widehat{U}_M^\dagger(\tau) |f\alpha_1\rangle \}. \end{aligned} \quad (14)$$

In the second-order approximation we obtain the expression for the jump probability $W(i\alpha \rightarrow f\alpha_1)$. The jump probability consists from three parts.

$$W(i\alpha \rightarrow f\alpha_1) = W_F(i\alpha \rightarrow f\alpha_1) + W_M(i\alpha \rightarrow f\alpha_1) + W_{\text{Int}}(i\alpha \rightarrow f\alpha_1), \quad (15)$$

where W_F is the probability of the jump during the free evolution, W_M is the probability of the jump during the measurement and W_{Int} is an interference term. The expressions for these probabilities are (see Refs. [11, 17] for the analogy of the derivation)

$$\begin{aligned} W_F(i\alpha \rightarrow f\alpha_1) & = \frac{1}{\hbar^2} \int_0^{T-\tau} dt_1 \int_0^{T-\tau} dt_2 \\ & \quad \times V(t_1 + t_0 + \tau)_{f\alpha_1, i\alpha} V(t_2 + t_0 + \tau)_{i\alpha, f\alpha_1} \\ & \quad \times \exp(i\omega_{f\alpha_1, i\alpha}(t_1 - t_2)), \end{aligned} \quad (16)$$

$$\begin{aligned} W_M(i\alpha \rightarrow f\alpha_1) & = \frac{1}{\hbar^2} \int_0^\tau dt_1 \int_0^\tau dt_2 \\ & \quad \times V(t_1 + t_0)_{f\alpha_1, i\alpha} V(t_2 + t_0)_{i\alpha, f\alpha_1} \\ & \quad \times \exp(i\omega_{f\alpha_1, i\alpha}(t_1 - t_2)) F(\lambda\omega_{fi}(t_1 - t_2)), \end{aligned} \quad (17)$$

$$\begin{aligned} W_{\text{Int}}(i\alpha \rightarrow f\alpha_1) & = \frac{2}{\hbar^2} \text{Re} \int_0^\tau dt_1 \int_\tau^T dt_2 \\ & \quad \times V(t_1 + t_0)_{f\alpha_1, i\alpha} V(t_2 + t_0)_{i\alpha, f\alpha_1} \\ & \quad \times \exp(i\omega_{f\alpha_1, i\alpha}(t_1 - t_2)) F(\lambda\omega_{if}(\tau - t_1)), \end{aligned} \quad (18)$$

where

$$\omega_{fi} = \frac{1}{\hbar}(E_f - E_i), \quad (19)$$

$$\omega_{f\alpha_1, i\alpha} = \omega_{fi} + \frac{E_1(f, \alpha_1) - E_1(i, \alpha)}{\hbar}, \quad (20)$$

$$F(x) = \langle \Phi | \exp(ix\hat{q}) | \Phi \rangle. \quad (21)$$

The probability to remain for the system in the initial state $|i\alpha\rangle$ is

$$W(i\alpha) = 1 - \sum_{f, \alpha_1} W(i\alpha \rightarrow f\alpha_1). \quad (22)$$

After N measurements the probability for the system to survive in the initial state is equal to $W(i\alpha)^N \approx$

$\exp(-RNT)$, where R is the measurement-modified decay rate

$$R = \sum_{f,\alpha_1} \frac{1}{T} W(i\alpha \rightarrow f\alpha_1). \quad (23)$$

4. Example

As an example we will consider the evolution of the measured two-level system. The system is forced by the periodic of the frequency ω_L perturbation $V(t)$ which induces the jumps from one state to another. Such a system was used in the experiment by Itano et al. [6]. The Hamiltonian of this system is

$$\hat{H} = \hat{H}_0 + \hat{V}(t), \quad (24)$$

where

$$\hat{H}_0 = \frac{\hbar\omega}{2} \hat{\sigma}_3, \quad (25)$$

$$\hat{V}(t) = (v\hat{\sigma}_+ + v^*\hat{\sigma}_-) \cos(\omega_L t). \quad (26)$$

Here $\sigma_1, \sigma_2, \sigma_3$ are Pauli matrices and $\sigma_{\pm} = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$. The Hamiltonian \hat{H}_0 has two eigenfunctions $|0\rangle$ and $|1\rangle$ with the eigenvalues $-\hbar\frac{\omega}{2}$ and $\hbar\frac{\omega}{2}$, respectively.

Using Eqs. (16), (17) and (18) for the jump from the state $|0\rangle$ to the state $|1\rangle$ we obtain

$$W_F(0 \rightarrow 1) = \frac{|v|^2 \sin^2\left(\frac{\Delta\omega}{2}(T - \tau)\right)}{\hbar^2 (\Delta\omega)^2}, \quad (27)$$

$$W_M(0 \rightarrow 1) = \frac{\tau |v|^2}{2 \hbar^2} \operatorname{Re} \int_0^{\tau} dt F(\lambda\omega t) \times \exp(i\Delta\omega t) \left(1 - \frac{t}{\tau}\right), \quad (28)$$

$$W_{\text{Int}}(0 \rightarrow 1) = \frac{|v|^2}{2\hbar^2} \operatorname{Re} \int_0^{\tau} dt_1 \int_{\tau}^T dt_2 \exp(i\Delta\omega(t_1 - t_2)) \times F(\lambda\omega(t_1 - \tau)), \quad (29)$$

where $\Delta\omega = \omega - \omega_L$ is the detuning. Equation (28) has been obtained in Ref. [11].

When λ is large, the function F varies rapidly and we can approximate expressions (28) and (29) as

$$W_M(0 \rightarrow 1) = \frac{\tau |v|^2}{2\Lambda\omega \hbar^2}, \quad (30)$$

$$W_{\text{Int}}(0 \rightarrow 1) = \frac{|v|^2}{\hbar^2} \frac{1}{2\Lambda\omega\Delta\omega} \sin(\Delta\omega(T - \tau)), \quad (31)$$

where $\Lambda = \lambda/C$, C is the width of the function F , defined by the equation (see Ref. [11])

$$C = \frac{1}{2} \int_{-\infty}^{\infty} F(x) dx \quad (32)$$

If $T \gg \tau$ and $\Delta\omega T \ll 1$ then we obtain

$$W(0 \rightarrow 1) = \frac{|v|^2 T^2}{\hbar^2 4} + \frac{|v|^2 T}{\hbar^2 2} \left(\frac{1}{\Lambda\omega} - \tau\right). \quad (33)$$

From Eq. (33) we see that the jump probability for the non-ideal measurement consists of two terms. The first term equals to the jump probability when the measurement is instantaneous, the second term represents the correction due to the finite duration of the measurement. In Ref. [11] it has been shown that the duration of the measurement can be estimated as

$$\tau \gtrsim \frac{1}{\Lambda\omega}. \quad (34)$$

From Eq. (33) we see that the correction term is small, since the duration of the measurement τ is almost compensated by the term $1/\Lambda\omega$.

5. Conclusion

The quantum Zeno effect is often analysed using the succession of the instantaneous measurements with free evolution of the measured system between the measurements. We analyze here the measurements with finite duration, instead. We apply the model of the measurement, developed in Ref. [11]. The equations for the jump probability (15)–(18) are obtained. Applying the equations to the measured two-level system we obtain a simple expression for the probability of the jump from one level to the other (33). The influence of the finite duration of the measurement is expressed as the small correction.

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