Influence of the detector's temperature on the quantum Zeno effect

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In this paper we study the quantum Zeno effect using the irreversible model of the measurement. The detector is modeled as a harmonic oscillator interacting with the environment. The oscillator is subjected to the force, proportional to the energy of the measured system. We use the Lindblad-type master equation to model the interaction with the environment. The influence of the detector's temperature on the quantum Zeno effect is obtained. It is shown that the quantum Zeno effect becomes stronger (the jump probability decreases) when the detector's temperature increases.

I. INTRODUCTION

The quantum Zeno effect is a consequence of the influence of the measurements on the evolution of a quantum system. In quantum mechanics the short-time behavior of the nondecay probability of unstable particles is not exponential but quadratic [1]. This deviation from the exponential decay has been observed by Wilkinson *et al.* [2]. In 1977, Misra and Sudarshan [3] showed that this behavior when combined with the quantum theory of measurement, based on the assumption of the collapse of the wave function, led to a very surprising conclusion: frequent observations slowed down the decay. An unstable particle would never decay when continuously observed. Misra and Sudarshan have called this effect the quantum Zeno paradox or effect. Later it was realized that the repeated measurements could not only slow the quantum dynamics, but the quantum process may be accelerated by frequent measurements as well. This effect was called a quantum anti-Zeno effect [4–6].

The quantum Zeno effect has been experimentally proved [7] in a repeatedly measured two-level system undergoing Rabi oscillations. The interruption of Rabi oscillations has been at the focus of interest [4,8–14]. Recently, the quantum Zeno effect has been considered for tunneling from a potential well into the continuum [15], as well as for photoionization [16]. The quantum anti-Zeno effect has been obtained experimentally [17].

In the analysis of the quantum Zeno effect the finite duration of the measurement becomes important and, therefore, the projection postulate is not sufficient to solve this problem. In Ref. [6] a simple model that allows to take into account the finite duration and finite accuracy of the measurement has been developed. However, this model does not take into account the irreversibility of the measurement process.

The basic ideas of a quantum measurement process were theoretically expounded in Refs. [18–24] on the assumption of environmentally induced decoherence or superselection. In this paper we extend the model, used in Ref. [6], including the interaction of the detector with the environment. Then it becomes possible to study the influence of other parameters

of the detector on the evolution of the measured system, too. In this paper we analyze the influence of the detector's temperature on the measured system.

To describe the decoherence and dissipation we use the the Lindblad-type master equation. Semigroup theories pioneered by Lindblad [25] demonstrated that the density-matrix positivity, translational invariance, and approach to thermal equilibrium cannot be satisfied simultaneously. Under the assumption of Markovian dynamics and the initial decoupling of the system and bath, the semigroup approach adds dissipative dynamics to the quantum master equations by means of the Lindblad dissipation operators.

Recently the semigroup formalism has attracted much attention. Quantum computing is one of the fields in which quantum dissipation finds the most recent applications. In physical chemistry, semigroup theories have been utilized to model dynamics of ultrafast predissociation in a condensed-phase or cluster environment [26], and electronic quenching due to the coupling of the adsorbate negative ion in resonance to the metal electrons in the desorption of neutral molecules on metal surfaces [27]. In nuclear physics, the semigroup formalism is applied to model giant resonances in the nuclear spectra above the neutron emission threshold [28].

We proceed as follows. In Sec. II we present the model of the measurement. The method of the solution is presented in Sec. III. The measurement of the unperturbed system is considered in Sec. IV. In Sec. V we derived a formula for the probability of the jump into another level during the measurement of the frequently measured perturbed system. Section VI summarizes our findings.

II. MODEL OF THE MEASUREMENT

We consider a system that consists of two parts. The first part of the system has the discrete energy spectrum. The Hamiltonian of this part is $\hat{H}_0$. The other part of the system is represented by Hamiltonian $\hat{H}_1$. Hamiltonian $\hat{H}_1$ commutes with $\hat{H}_0$. In a particular case the second part can be absent and $\hat{H}_1$ can be zero. The operator $\hat{V}(t)$ causes the jumps between different energy levels of $\hat{H}_0$. Therefore, the full Hamiltonian of the system is of the form $\hat{H}_S = \hat{H}_0 + \hat{H}_1 + \hat{V}(t)$. The example of such a system is an atom with the Hamiltonian $\hat{H}_0$ interacting with the electromagnetic field,

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represented by $\hat{H}_1$, while the interaction between the atom and the field is $\hat{V}(t)$.

We will measure in which eigenstate of the Hamiltonian $\hat{H}_0$ the system is. The measurement is performed by coupling the system with the detector. The full Hamiltonian of the system and the detector is

$$\hat{H} = \hat{H}_S + \hat{H}_D + \hat{H}_I, \tag{1}$$

where $\hat{H}_D$ is the Hamiltonian of the detector and $\hat{H}_I$ represents the interaction between the detector and the measured system, described by the Hamiltonian $\hat{H}_0$. As the detector we use a harmonic oscillator with the Hamiltonian

$$\hat{H}_D = \hbar\Omega(\hat{b}^\dagger\hat{b} + \tfrac{1}{2}), \tag{2}$$

where $\hat{b}$ and $\hat{b}^\dagger$ are the creation and anihillation operators, respectively. We choose the interaction operator $\hat{H}_I$ in the form

$$\hat{H}_I = \lambda\hat{q}\hat{H}_0, \tag{3}$$

where $\hat{q} = \hat{b}^\dagger + \hat{b}$ is the coordinate of the detector and the parameter λ describes the strength of the interaction. This system-detector interaction is similar to that considered by von Neumann [29] and in Refs. [6,30–34]. In order to obtain a sensible measurement, the parameter λ must be large.

The measurement begins at time moment t_0. At the beginning of the interaction with the detector, the detector's density matrix is $\hat{\rho}_D(t_0)$. The detector initially is in the thermal equilibrium with the temperature T. Therefore,

$$\hat{\rho}_D(t_0) = \hat{\rho}_T = \exp\left(-\frac{\hbar\Omega\hat{n}}{k_BT}\right)\left[1 - \exp\left(-\frac{\hbar\Omega}{k_BT}\right)\right], \tag{4}$$

where $\hat{n} = \hat{b}^\dagger\hat{b}$. The average excitation of the detector in thermal equilibrium with the temperature T is

$$\bar{n}(T) = \left[\exp\left(\frac{\hbar\Omega}{k_BT}\right) - 1\right]^{-1}. \tag{5}$$

The full density matrix of the system and detector is $\hat{\rho}(t_0) = \hat{\rho}_S(t_0) \otimes \hat{\rho}_D(t_0)$, where $\hat{\rho}_S(t_0)$ is the density matrix of the system.

The detector is interacting with the environment. The master equation for the density matrix of the system and the detector in the Lindblad form is (Ref. [25])

$$\frac{\partial\hat{\rho}(t)}{\partial t} = \frac{1}{i\hbar}[\hat{H},\hat{\rho}(t)] + L_D[\hat{\rho}(t)], \tag{6}$$

where

$$L_D[\hat{\rho}(t)] = \sum_\mu ([\hat{V}_\mu\hat{\rho}(t),\hat{V}_\mu^\dagger] + [\hat{V}_\mu,\hat{\rho}(t)\hat{V}_\mu^\dagger]), \tag{7}$$

and $\hat{V}_\mu$ are the Lindblad dissipation operators. We use the equation of a dissipative phase-damped oscillator discussed in the quantum optic [18]. The Lindblad dissipation operators are chosen as follows:

$$\hat{V}_1 = \sqrt{\frac{\gamma}{2}}\hat{a}^\dagger\hat{a}, \quad \hat{V}_2 = \sqrt{\frac{\gamma_\uparrow}{2}}\hat{a}^\dagger, \quad \hat{V}_3 = \sqrt{\frac{\gamma_\downarrow}{2}}\hat{a}. \tag{8}$$

Then Eq. (6) for the density matrix becomes

$$\frac{\partial\hat{\rho}(t)}{\partial t} = \frac{1}{i\hbar}[\hat{H},\hat{\rho}(t)] + \frac{\gamma}{2}[2\hat{n}\hat{\rho}(t)\hat{n} - \hat{n}^2\hat{\rho}(t) - \hat{\rho}(t)\hat{n}^2]$$

$$+ \frac{\gamma_\uparrow}{2}[2\hat{a}^\dagger\hat{\rho}(t)\hat{a} - (\hat{n}+1)\hat{\rho}(t) - \hat{\rho}(t)(\hat{n}+1)]$$

$$+ \frac{\gamma_\downarrow}{2}[2\hat{a}\hat{\rho}(t)\hat{a}^\dagger - \hat{n}\hat{\rho}(t) - \hat{\rho}(t)\hat{n}]. \tag{9}$$

The approach thermal equilibrium is obtained when the parameters $\gamma_\uparrow$ and $\gamma_\downarrow$ satisfy the condition [35]

$$\gamma_\uparrow = \gamma_\downarrow\exp\left(-\frac{\hbar\Omega}{k_BT}\right). \tag{10}$$

III. SOLUTION OF THE MASTER EQUATION

For the solution of Eq. (9) we adopt the technique used in Ref. [36]. We introduce the quantum characteristic function [37]

$$\chi(\xi,\xi^*) = \text{Tr}\{\hat{\rho}e^{\xi\hat{b}^\dagger}e^{-\xi^*\hat{b}}\}. \tag{11}$$

The quantum characteristic function of the detector at thermal equilibrium is

$$\chi_T(\xi,\xi^*) = \exp[-\xi\xi^*\bar{n}(T)]. \tag{12}$$

We multiply Eq. (6) by $\exp(-\xi^*\hat{b})$ from the left and by $\exp(\xi\hat{b}^\dagger)$ from the right and take the trace. When the interaction between the measured system and the detector is absent (i.e., $\lambda = 0$), we obtain the equation

$$\frac{\partial}{\partial t}\chi(\xi,\xi^*;t) = i\Omega\left(\xi\frac{\partial}{\partial\xi}\chi - \xi^*\frac{\partial}{\partial\xi^*}\chi\right) + \frac{\gamma}{2}\left(2\xi^*\xi\frac{\partial^2}{\partial\xi\partial\xi^*}\chi\right.$$

$$\left. - \xi^{*2}\frac{\partial^2}{\partial\xi^{*2}}\chi - \xi^2\frac{\partial^2}{\partial\xi^2}\chi - \xi^*\frac{\partial}{\partial\xi^*}\chi - \xi\frac{\partial}{\partial\xi}\chi\right)$$

$$+ \frac{\gamma_\uparrow}{2}\left(\xi\frac{\partial}{\partial\xi}\chi + \xi^*\frac{\partial}{\partial\xi^*}\chi - 2\xi\xi^*\chi\right)$$

$$- \frac{\gamma_\downarrow}{2}\left(\xi\frac{\partial}{\partial\xi}\chi + \xi^*\frac{\partial}{\partial\xi^*}\chi\right). \tag{13}$$

We will search for a solution of Eq. (13) in the form

$$\chi(\xi,\xi^*)=\exp\left(\sum_{j,k} C_{j,k}(t)\xi^j(-\xi^*)^k\right), \qquad (14)$$

where $C_{j,k}$ are the coefficients to be determined. Substituting Eq. (14) into Eq. (13) we obtain the set of equations for the coefficients $C_{j,k}$,

$$\frac{\partial C_{j,k}(t)}{\partial t}=i\Omega(j-k)C_{j,k}(t)-\frac{\gamma}{2}(j-k)^2C_{j,k}(t)+\frac{1}{2}(\gamma_\uparrow-\gamma_\downarrow)$$

$$\times(j+k)C_{j,k}(t)+\gamma_\uparrow\delta_{j,1}\delta_{k,1}. \qquad (15)$$

The solution of Eq. (15) is

$$C_{1,1}(t)=C_{1,1}(0)e^{-(\gamma_\downarrow-\gamma_\uparrow)t}+\bar{n}(T)(1-e^{-(\gamma_\downarrow-\gamma_\uparrow)t}), \qquad (16)$$

$$C_{j,k}(t)=C_{j,k}(0)e^{i\Omega(j-k)t}e^{-(\gamma/2)(j-k)^2t-(1/2)(\gamma_\downarrow-\gamma_\uparrow)(j+k)t},$$

$$j\neq1, \quad k\neq1. \qquad (17)$$

From the solution we see that the function χ approaches the function at the equilibrium χ_T as the time t grows. The detector's density matrix $\widehat{\rho_D}$, correspondingly, tends to $\widehat{\rho_T}$.

IV. MEASUREMENT OF THE UNPERTURBED SYSTEM

At first, we will consider the case when the perturbation is absent, i.e., $\hat{V}(t)=0$. Since the Hamiltonian of the measured system does not depend on t we will omit the parameter t_0 in this section. We can choose the basis $|n\alpha\rangle$ common for the operators $\hat{H}_0$ and $\hat{H}_1$,

$$\hat{H}_0|n\alpha\rangle=E_n|n\alpha\rangle, \qquad (18)$$

$$\hat{H}_1|n\alpha\rangle=E_1(n,\alpha)|n\alpha\rangle, \qquad (19)$$

where n numbers the eigenvalues of the Hamiltonian $\hat{H}_0$ and α represents the remaining quantum numbers.

We introduce the density matrix $\hat{\rho}_{m,n}=\Sigma_\alpha\langle m\alpha|\hat{\rho}|n\alpha\rangle$ and the characteristic function

$$\chi_{m,n}(\xi,\xi^*;t)=\text{Tr}\{\hat{\rho}_{m,n}(t)e^{\xi\hat{b}^\dagger}e^{-\xi^*\hat{b}}\}. \qquad (20)$$

From Eq. (6) we obtain the equation for the density matrix $\hat{\rho}_{m,n}$,

$$\frac{\partial}{\partial t}\hat{\rho}_{m,n}=i\omega_{mn}\hat{\rho}_{m,n}-i\Omega(\hat{n}\hat{\rho}_{m,n}-\hat{\rho}_{m,n}\hat{n})$$

$$-i\lambda(\omega_m\hat{q}\hat{\rho}_{m,n}-\hat{\rho}_{m,n}\hat{q}\omega_n)+L_D[\hat{\rho}_{m,n}], \qquad (21)$$

where

$$\omega_n=\frac{E_n}{\hbar}, \qquad (22)$$

$$\omega_{mn}=\omega_m-\omega_n. \qquad (23)$$

Equation (21) may be solved similarly as in Sec. III. When the detector is initially at equilibrium, then $\chi_{mn}(\xi,\xi^*;0)=\chi_{mn}(0)\exp[-\xi\xi^*\bar{n}(T)]$. As in Sec. III we take the characteristic function of the form (14) and obtain the equations for the coefficients $C_{j,k}$,

$$\frac{\partial C_{0,0}}{\partial t}=-i\omega_{mn}[1+\lambda(C_{1,0}+C_{0,1})], \qquad (24)$$

$$\frac{\partial C_{1,0}}{\partial t}=(i\Omega-\gamma_{\text{eff}}]C_{1,0}+i\lambda(\omega_n-\omega_{mn}C_{1,1}), \qquad (25)$$

$$\frac{\partial C_{0,1}}{\partial t}=-(i\Omega+\gamma_{\text{eff}})C_{0,1}-i\lambda(\omega_m+\omega_{mn}C_{1,1}), \qquad (26)$$

$$\frac{\partial C_{1,1}}{\partial t}=(\gamma_\uparrow-\gamma_\downarrow)C_{1,1}+\gamma_\uparrow, \qquad (27)$$

with the initial conditions $C_{0,0}(0)=0,C_{1,0}(0)=0,C_{0,1}(0)=0,C_{1,1}(0)=\bar{n}(T)$. Here

$$\gamma_{\text{eff}}=\tfrac{1}{2}(\gamma+\gamma_\downarrow-\gamma_\uparrow). \qquad (28)$$

The solutions of Eqs. (24)–(27) are

$$C_{1,1}(t)=\bar{n}(T), \qquad (29)$$

$$C_{1,0}(t)=i\lambda\frac{\omega_n-\omega_{mn}\bar{n}(T)}{\gamma_{\text{eff}}-i\Omega}(1-e^{(i\Omega-\gamma_{\text{eff}})t}), \qquad (30)$$

$$C_{0,1}(t)=-i\lambda\frac{\omega_m+\omega_{mn}\bar{n}(T)}{\gamma_{\text{eff}}+i\Omega}(1-e^{-(i\Omega+\gamma_{\text{eff}})t}), \qquad (31)$$

$$C_{0,0}(t)=-i\omega_{mn}t+\lambda^2\omega_{mn}\frac{\omega_n-\omega_{mn}\bar{n}(T)}{\gamma_{\text{eff}}-i\Omega}$$

$$\times\left(t+\frac{1}{\gamma_{\text{eff}}-i\Omega}(e^{(i\Omega-\gamma_{\text{eff}})t}-1)\right)$$

$$-\lambda^2\omega_{mn}\frac{\omega_m+\omega_{mn}\bar{n}(T)}{\gamma_{\text{eff}}+i\Omega}$$

$$\times\left(t+\frac{1}{\gamma_{\text{eff}}+i\Omega}(e^{-(i\Omega+\gamma_{\text{eff}})t}-1)\right). \qquad (32)$$

Using Eqs. (14) and (32) we find that the nondiagonal elements of the density matrix of the measured system become small as the time t grows. This represents the decoherence induced by the measurement. The diagonal elements of the density matrix do not change.

V. MEASUREMENT OF THE PERTURBED SYSTEM

The operator $\hat{V}(t)$ represents the perturbation of the unperturbed Hamiltonian $\hat{H}_0+\hat{H}_1$. We will take into account the influence of the operator $\hat{V}$ by the perturbation method,

assuming that the strength of the interaction λ between the system and detector is large.

The density matrix at time t is related to the initial density matrix by the equation $\hat{\rho}(t) = S(t)\hat{\rho}(0)$. The superoperator S obeys the equation

$$\frac{\partial}{\partial t} S = LS, \tag{33}$$

where the Liouvillian superoperator L is defined by the equation

$$L\hat{\rho} = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}] + L_D[\hat{\rho}]. \tag{34}$$

Here $\hat{H}$ and L_D are defined by Eqs. (1) and (7), respectively. We can write $L = L_0 + L_V$, where L_V is a small perturbation, defined by the equation

$$L_V\hat{\rho} = \frac{1}{i\hbar}[\hat{V}, \hat{\rho}]. \tag{35}$$

We expand the superoperator S into powers of V

$$S = S^{(0)} + S^{(1)} + S^{(2)} + \cdots.$$

Then from Eq. (33) follows

$$\frac{\partial}{\partial t} S^{(0)} = L_0 S^{(0)}, \tag{36}$$

$$\frac{\partial}{\partial t} S^{(i)} = L_0 S^{(i)} + L_V S^{(i-1)}. \tag{37}$$

The formal solutions of Eqs. (36) and (37) are

$$S^{(0)} = e^{L_0 t} \tag{38}$$

and

$$S^{(i)} = \int_0^t dt_1 S^{(0)}(t - t_1) L_V S^{(i-1)}(t_1). \tag{39}$$

In the second-order approximation we have

$$S(t) = S^{(0)}(t) + \int_0^t dt_1 S^{(0)}(t - t_1) L_V S^{(0)}(t_1)$$

$$+ \int_0^t dt_1 \int_0^{t_1} dt_2 S^{(0)}(t - t_1) L_V S^{(0)}(t_1 - t_2) L_V S^{(0)}(t_2). \tag{40}$$

Let the initial density matrix of the system and detector be

$$\hat{\rho}(0) = |i\alpha\rangle\langle i\alpha| \otimes \hat{\rho}_D, \tag{41}$$

where $\hat{\rho}_D$ is the density matrix of the detector. The probability of the jump from the level $|i\alpha\rangle$ to the level $|f\alpha_1\rangle$ during the measurement is

$$W(i\alpha \rightarrow f\alpha_1, t) = \text{Tr}\{|f\alpha_1\rangle\langle f\alpha_1|\hat{\rho}(t)\}. \tag{42}$$

The unperturbed evolution does not change the energy of the measured system, therefore, we can write

$$S^{(0)}(t)[|m\alpha\rangle\langle n\alpha'| \otimes \hat{\rho}_D] = |m\alpha\rangle\langle n\alpha'| \otimes S^{(0)}_{m\alpha,n\alpha'}(t)\hat{\rho}_D. \tag{43}$$

Equation (43) defines a new superoperator $S^{(0)}_{m\alpha,n\alpha'}$ acting only on the density matrix of the detector. The indices $m\alpha$ and $n\alpha'$ in $S^{(0)}_{m\alpha,n\alpha'}$ denote the states of the measured system. From Eq. (43) it follows that the superoperator $S^{(0)}_{m\alpha,m\alpha}$ with equal indices does not change the trace of the density matrix $\hat{\rho}_D$, since the trace of the full density matrix of the measured system and the detector must remain unchanged during the evolution.

We assume that diagonal matrix elements of the perturbation operator V are zeros. Inserting the expression $\hat{\rho}(t) = S(t)\hat{\rho}(0)$ into Eq. (42) and using Eq. (40) for the superoperator $S(t)$, we obtain the jump probability

$$W(i\alpha \rightarrow f\alpha_1, t)$$

$$= \frac{1}{\hbar^2} \int_0^t dt_1 \int_0^{t_1} dt_2 \text{Tr}\{|f\alpha_1\rangle\langle f\alpha_1|(S^{(0)}(t - t_1)\hat{V}S^{(0)}$$

$$\times (t_1 - t_2)[S^{(0)}(t_2)\hat{\rho}(0)]\hat{V} + S^{(0)}(t - t_1)$$

$$\times [S^{(0)}(t_1 - t_2)\hat{V}S^{(0)}(t_2)\hat{\rho}(0)]\hat{V} - S^{(0)}(t - t_1)\hat{V}S^{(0)}$$

$$\times (t_1 - t_2)\hat{V}S^{(0)}(t_2)\hat{\rho}(0) - S^{(0)}(t - t_1)\{S^{(0)}(t_1 - t_2)$$

$$\times [S^{(0)}(t_2)\hat{\rho}(0)]\hat{V}\}\hat{V})\}. \tag{44}$$

From Eqs. (43) and (41) it follows that the last two terms in Eq. (44) contain the scalar product $\langle f\alpha_1|i\alpha\rangle$. Since the states $|f\alpha_1\rangle$ and $|i\alpha\rangle$ are orthogonal, the last two terms in Eq. (44) are zeros. Therefore, the jump probability is

$$W(i\alpha \rightarrow f\alpha_1, t) = \frac{1}{\hbar^2} \int_0^t dt_1 \int_0^{t_1} dt_2 \text{Tr}(|f\alpha_1\rangle\langle f\alpha_1|$$

$$\times \{S^{(0)}(t - t_1)\hat{V}S^{(0)}(t_1 - t_2)$$

$$\times [S^{(0)}(t_2)\hat{\rho}(0)]\hat{V} + S^{(0)}(t - t_1)$$

$$\times [S^{(0)}(t_1 - t_2)\hat{V}S^{(0)}(t_2)\hat{\rho}(0)]\hat{V}). \tag{45}$$

From Eq. (45), using the expression for the initial density matrix of the system and the detector, Eq. (41), and Eq. (43), we have

$$W(i\alpha \rightarrow f\alpha_1,t) = \frac{1}{\hbar^2}|V_{i\alpha,f\alpha_1}|^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \text{Tr}\{S_{f\alpha_1,f\alpha_1}^{(0)}$$

$$\times (t-t_1)[S_{i\alpha,f\alpha_1}^{(0)}(t_1-t_2)$$

$$+ S_{f\alpha_1,i\alpha}^{(0)}(t_1-t_2)]S_{i\alpha,i\alpha}^{(0)}(t_2)\hat{\rho}_D\}. \tag{46}$$

The superoperator $S_{f\alpha_1,f\alpha_1}^{(0)}(t-t_1)$ preserves the trace of the detector's density matrix, therefore, the jump probability equals to

$$W(i\alpha \rightarrow f\alpha_1,t)$$

$$= \frac{1}{\hbar^2}|V_{i\alpha,f\alpha_1}|^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \text{Tr}\{[S_{i\alpha,f\alpha_1}^{(0)}$$

$$\times (t_1-t_2) + S_{f\alpha_1,i\alpha}^{(0)}(t_1-t_2)]S_{i\alpha,i\alpha}^{(0)}(t_2)\hat{\rho}_D\}. \tag{47}$$

Defining a new characteristic function similarly as in Eq. (20),

$$\chi_{i\alpha,f\alpha_1}(\xi,\xi^*;t_1,t_2) = \text{Tr}\{e^{\xi\hat{b}^\dagger}e^{-\xi^*\hat{b}}S_{i\alpha,f\alpha_1}^{(0)}$$

$$\times (t_1-t_2)S_{i\alpha,i\alpha}^{(0)}(t_2)\hat{\rho}_D\}, \tag{48}$$

the jump probability (47) can be expressed as

$$W(i\alpha \rightarrow f\alpha_1,t) = \frac{1}{\hbar^2}|V_{i\alpha,f\alpha_1}|^2 \int_0^t dt_1$$

$$\times \int_0^{t_1} dt_2 [\chi_{i\alpha,f\alpha_1}(0,0;t_1,t_2)$$

$$+ \chi_{f\alpha_1,i\alpha}(0,0;t_1,t_2)]. \tag{49}$$

The detector initially (at $t=0$) is in the thermal equilibrium with the temperature T, $\hat{\rho}_D = \hat{\rho}_T$ [Eq. (4)]. The initial characteristic function is $\chi_{i\alpha,f\alpha_1}(\xi,\xi^*;0,0) = \exp[-\xi\xi^*\bar{n}(T)]$. Using the results of the Sec. IV [Eqs. (14) and (29)–(32)], we obtain the characteristic function of the density matrix at time t_2,

$$\chi_{i\alpha,f\alpha_1}(\xi,\xi^*;t_2,t_2) = \text{Tr}\{e^{\xi\hat{b}^\dagger}e^{-\xi^*\hat{b}}S_{i\alpha,i\alpha}^{(0)}(t_2)\hat{\rho}_D\}$$

$$= \exp\left(\frac{i\xi\lambda\omega_i}{\gamma_{\text{eff}} - i\Omega}(1 - e^{i(\Omega - \gamma_{\text{eff}})t_2})\right)$$

$$\times \exp\left(\frac{i\xi^*\lambda\omega_i}{\gamma_{\text{eff}} + i\Omega}(1 - e^{-i(\Omega + \gamma_{\text{eff}})t_2})\right)$$

$$- \xi\xi^*\bar{n}(T)\right). \tag{50}$$

Taking the function $\chi_{i\alpha,f\alpha_1}(\xi,\xi^*;t_2,t_2)$ from Eq. (50) as the initial characteristic function and proceding further as in Sec. IV, we have the value of the characteristic function, defined by Eq. (48) with the parameters $\xi = \xi^* = 0$,

$$\chi_{i\alpha,f\alpha_1}(0,0;t_1,t_2) = \exp\left[-i\omega_{i\alpha,f\alpha_1}(t_1-t_2) + \lambda^2\omega_{if}\frac{\omega_f - \omega_{if}\bar{n}(T)}{\gamma_{\text{eff}} - i\Omega}\left(t_1 - t_2 + \frac{1}{\gamma_{\text{eff}} - i\Omega}(e^{(i\Omega - \gamma_{\text{eff}})(t_1-t_2)} - 1)\right)\right.$$

$$- \lambda^2\omega_{if}\frac{\omega_i + \omega_{if}\bar{n}(T)}{\gamma_{\text{eff}} + i\Omega}\left(t_1 - t_2 + \frac{1}{\gamma_{\text{eff}} + i\Omega}(e^{-(i\Omega + \gamma_{\text{eff}})(t_1-t_2)} - 1)\right) + \frac{\lambda^2\omega_{if}\omega_i}{(\gamma_{\text{eff}} - i\Omega)^2}(1 - e^{(i\Omega - \gamma_{\text{eff}})t_2})$$

$$\times (1 - e^{(i\Omega - \gamma_{\text{eff}})(t_1-t_2)}) - \frac{\lambda^2\omega_{if}\omega_i}{(\gamma_{\text{eff}} + i\Omega)^2}(1 - e^{-(i\Omega + \gamma_{\text{eff}})t_2})(1 - e^{-(i\Omega + \gamma_{\text{eff}})(t_1-t_2)})\right]. \tag{51}$$

Here

$$\omega_{i\alpha,f\alpha_1} = \omega_{if} + \frac{1}{\hbar}[E_1(i,\alpha) - E_1(f,\alpha_1)]. \tag{52}$$

Approximations

When the dissipation is fast, i.e., the dissipation time is much less than the period of the oscillator, we have $\Omega \ll \gamma_{\text{eff}}$. Then

$$\chi_{i\alpha,f\alpha_1}(0,0;t_1,t_2) = \exp[-i\omega_{i\alpha,f\alpha_1}(t_1-t_2)]$$

$$\times \exp\left[-[1 + 2\bar{n}(T)]\frac{\lambda^2\omega_{if}^2}{\gamma_{\text{eff}}}\right.$$

$$\times \left(t_1 - t_2 + \frac{1}{\gamma_{\text{eff}}}(e^{-\gamma_{\text{eff}}(t_1-t_2)} - 1)\right)\right].$$

The probability of the jump from the level $|i\alpha\rangle$ to the level $|f\alpha_1\rangle$ during the measurement according to Eq. (49) is

$$W(i\alpha \rightarrow f\alpha_1, t) = \frac{2t}{\hbar^2} |V_{i\alpha, f\alpha_1}|^2 \mathrm{Re} \int_0^t du \left(1 - \frac{u}{t}\right) e^{i\omega_{f\alpha_1, i\alpha} u}$$

$$\times \exp\left[-\frac{[1 + 2\bar{n}(T)]\lambda^2 \omega_{if}^2}{\gamma_{\mathrm{eff}}} \right.$$

$$\left. \times \left(u + \frac{1}{\gamma_{\mathrm{eff}}}(e^{-\gamma_{\mathrm{eff}} u} - 1)\right) \right]. \tag{53}$$

We introduce the function

$$\Phi(t)_{f\alpha_1, i\alpha} = |V_{i\alpha, f\alpha_1}|^2 \exp\left(\frac{i}{\hbar}[E_1(f, \alpha_1) - E_1(i, \alpha)]t\right) \tag{54}$$

and the Fourier transformation of $\Phi(t)_{f\alpha_1, i\alpha}$,

$$G(\omega)_{f\alpha_1, i\alpha} = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \Phi(t)_{f\alpha_1, i\alpha} \exp(-i\omega t). \tag{55}$$

Then we can rewrite Eq. (53) in the form

$$W(i\alpha \rightarrow f\alpha_1, t) = \frac{2\pi t}{\hbar^2} \int_{-\infty}^{\infty} d\omega G(\omega)_{f\alpha_1, i\alpha} P(\omega)_{if}, \tag{56}$$

where

$$P(\omega)_{if} = \frac{1}{\pi} \mathrm{Re} \int_0^t du \left(1 - \frac{u}{t}\right) \exp[i(\omega - \omega_{if})u]$$

$$\times \exp\left[-\frac{[1 + 2\bar{n}(T)]\lambda^2 \omega_{if}^2}{\gamma_{\mathrm{eff}}} \right.$$

$$\left. \times \left(u + \frac{1}{\gamma_{\mathrm{eff}}}(e^{-\gamma_{\mathrm{eff}} u} - 1)\right) \right]. \tag{57}$$

Equation (56) is of the form obtained by Kofman and Kurizki [5], assuming the ideal instantaneous projections. The function $P(\omega)_{if}$ is the measurement-modified shape of the spectral line (Refs. [5,6,38]). Here we have shown that Eq. (56) can be derived from a more realistic model as well. Due to the assumption that dissipation is fast, $\Omega \ll \gamma_{\mathrm{eff}}$ is crucial. Without this assumption the jump probability cannot have

the form of Eq. (56), since then $\chi_{i\alpha, f\alpha_1}(0,0;t_1,t_2)$ depends not only on the difference $t_1 - t_2$ but also on t_2.

When λ is big, the integral in Eq. (53) contributes only small values of u and we can expand the exponent $\times \exp(-\gamma_{\mathrm{eff}} u)$ into a Taylor series keeping the first three terms only. We obtain the jump rate

$$R(i\alpha \rightarrow f\alpha_1) \approx \frac{2}{\hbar^2} |V_{i\alpha, f\alpha_1}|^2 \mathrm{Re} \int_0^{\infty} du \exp\left(i\omega_{f\alpha_1, i\alpha} u \right.$$

$$\left. -\frac{1}{2}[1 + 2\bar{n}(T)]\lambda^2 \omega_{if}^2 u^2 \right)$$

or

$$R(i\alpha \rightarrow f\alpha_1) \approx \frac{2|V_{i\alpha, f\alpha_1}|^2}{\hbar^2 \lambda |\omega_{if}|} \sqrt{\frac{\pi}{2[1 + 2\bar{n}(T)]}}. \tag{58}$$

The obtained decay rate is inversely proportional to the measurement strength λ. The measurement strength appears in the equations multiplied by $\sqrt{1 + 2\bar{n}(T)}$, therefore, the effect of the measurement increases as the temperature of the detector grows.

VI. CONCLUSIONS

We analyze the quantum Zeno effect using the irreversible model of the measurement. The detector is modeled as a harmonic oscillator, initially being at the thermal equilibrium. The interaction of the detector with the system is modeled similarly as in Ref. [6]. The Lindblad-type master equation for the detectors density matrix is solved analytically. An equation for the probability of the jump between the measured system's states during the measurement, similar to that of Refs. [5,6,38], is obtained Eq. (56). From the used model it follows that the increase of the detector's temperature leads to the enhancement of the quantum Zeno or quantum anti-Zeno effects.

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