Non-Abelian Gauge Potentials for Ultracold Atoms with Degenerate Dark States

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We show that the adiabatic motion of ultracold, multilevel atoms in spatially varying laser fields can give rise to effective non-Abelian gauge fields if degenerate adiabatic eigenstates of the atom-laser interaction exist. A pair of such degenerate dark states emerges, e.g., if laser fields couple three internal states of an atom to a fourth common one under pairwise two-photon-resonance conditions. For this so-called tripod scheme we derive general conditions for truly non-Abelian gauge potentials and discuss special examples. In particular we show that using orthogonal laser beams with orbital angular momentum an effective magnetic field can be generated that has a monopole component.

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Gauge fields are a central building block of the theory of fundamental interactions. As dynamical variables they are responsible for the forces between elementary particles. On the other hand, also nondynamical, i.e., prescribed gauge fields, are of interest in a variety of single- and many-body quantum systems. For example, an external magnetic field applied to a gas of noninteracting electrons can lead to the integer quantum Hall effect [1]. In the presence of a lattice potential, the eigenenergies of the lowest Bloch band form a fractal structure depending on the magnetic flux that passes through the unit cell [2]. If, in addition, there are strong interactions between the particles as, e.g., in a two-dimensional electron gas subject to a magnetic field, fractional quantum Hall structures [3] and Laughlin liquids [4] can emerge.

In recent years ultracold atomic gases [5] have become an ideal playground to experimentally investigate many-body physics. This is due to their enormous versatility and the advanced experimental techniques available in atomic and optical physics. One of the most fascinating subjects in this context is the study of the effects of artificial magnetic fields [6]. To create an artificial magnetic field for neutral atoms one can, e.g., rotate the trapping potential confining the atoms. This feasible but challenging approach is currently pursued in several laboratories [7]. An alternative is based on the adiabatic motion of Λ-type 3-level atoms in laser fields that create a nondegenerate dark state. If the dark state of the atom is space dependent, the motion of atoms adiabatically following it is associated with a topological or Berry phase [8,9]. A proper description of such a motion naturally leads to gauge potentials [9–11]. As shown in [12,13] an effective magnetic field can arise, e.g., if Λ-type atoms interact with pairs of laser fields that possess a relative orbital angular momentum. The advantage of this scheme as compared to rotating traps is that it is not limited to rotationally symmetric configurations. Furthermore, in the rotating traps only a constant effective magnetic field is created [7], whereas using optical means the effective magnetic field can be controlled and shaped [13]. The description of the adiabatic motion of atoms in terms of gauge potentials has been generalized to $j + 1 \rightarrow j$ transitions in [14]. The effects of gauge potentials on strongly interacting, bosonic atoms in one-dimensional optical lattices have been analyzed in [15], where it was shown that they lead to interesting modifications of the Bose-Hubbard model. An alternative way to create artificial magnetic fields in lattice gases was recently suggested employing laser assisted, state-dependent tunneling [16,17] or oscillating potentials with spatial modulations [18]. In all of these systems the gauge fields have, however, $\mathsf{U}(1)$ symmetry; i.e., they are *Abelian.*

As shown by Wilczek and Zee, *non-Abelian* gauge fields can arise in the adiabatic dynamics of quantum systems with multiple degenerate eigenstates [19]. One of the interesting properties of non-Abelian gauge potentials is the possibility of magnetic monopoles. The presence of effective magnetic monopole fields in simple quantum systems was first pointed out by Moody, Shapere, and Wilczek discussing the adiabatic nuclear rotation in a diatomic molecule [20]. In this Letter we propose an experimentally realizable scheme that allows one to study the motion and the interaction of neutral quantum gases in non-Abelian gauge fields. We show, in particular, that the coupling of multilevel atoms to spatially varying laser fields can give rise to such potentials for the atomic center-of-mass motion. A necessary condition for this is that the atom-laser interaction has degenerate dark eigenstates with a nonvanishing nonadiabatic coupling.

Gauge structures in atomic systems with multiple degenerate dark states have first been discussed by Visser and Nienhuis [14] considering atoms with a $j + 1 \rightarrow j$ $(j > 1)$ transition driven by circularly polarized laser light, as shown in Fig. 1(a). Since in such a scheme the dark states are exactly decoupled, the associated gauge potentials again have, however, $\mathsf{U}(1)$ symmetry. The simplest system with a nonvanishing adiabatic coupling between degener-

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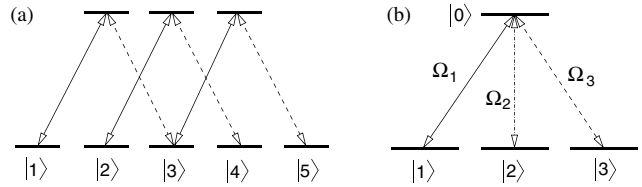

FIG. 1. (a) $j = 2 \rightarrow j = 1$ transition with two degenerate dark states in the manifolds $\{|1\rangle, |3\rangle, |5\rangle\}$ and $\{|2\rangle, |4\rangle\}$ which are not coupled by nonadiabatic transitions. (b) Tripod-coupling scheme forming two degenerate dark states with nonadiabatic coupling.

ate dark states is the so-called tripod scheme shown in Fig. 1(b) [21]. For this scheme the possibility of non-Abelian topological phases in the interaction of an atom with appropriately pulsed laser fields has been predicted in [22]. Furthermore, applications to geometric quantum computation have been put forward and investigated in [23,24].

After a general discussion of non-Abelian gauge potentials in the adiabatic motion of atoms in laser fields, we introduce the tripod-coupling scheme as the simplest system leading to non-Abelian gauge fields. We then discuss specific examples. In particular, we show that, using orthogonal laser beams with orbital angular momentum, an effective magnetic field can be generated that has a monopole component. It should be noted that nonlinear interactions in multicomponent spinor Bose-Einstein condensates can allow for excitations which themselves have a nontrivial topological structure like Skyrmions [25] or monopoles [26–28]. We consider here, however, the motion and interaction of atoms in external non-Abelian gauge fields.

We start by extending the discussion of Wilczek and Zee [19] and Moody, Shapere, and Wilczek [20] to the adiabatic motion of atoms in stationary laser fields. For this we consider atoms with multiple internal states. For fixed position $\mathbf{r}$ the internal Hamiltonian $\hat{H}_0(\mathbf{r})$ including the laser interaction can be diagonalized to give a set of, say, N dressed states $|\chi_n(\mathbf{r})\rangle$ with eigenvalues $\varepsilon_n(\mathbf{r})$, where $n = 1, 2, \ldots, N$. The full quantum state of the atom describing both internal and motional degrees of freedom can then be expanded in terms of the dressed states according to $|\Phi\rangle = \Sigma_{n=1}^{N} \Psi_n(\mathbf{r})|\chi_n(\mathbf{r})\rangle$. The N-dimensional column vector of wave functions $\Psi = (\Psi_1, \Psi_2, \ldots, \Psi_N)^\top$ obeys the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\Psi = \left[\frac{1}{2m}(-i\hbar\nabla - \mathbf{A})^2 + V\right]\Psi, \tag{1}$$

m being the mass of the atoms, and V being an external potential that confines the motion of atoms to a finite region in space. Here V and $\mathbf{A}$ are $N \times N$ matrices, the latter appearing due to the position dependence of the atomic dressed states:

$$V_{n,m} = \varepsilon_n(\mathbf{r})\delta_{n,m} + \langle\chi_n(\mathbf{r})|V(\mathbf{r})|\chi_m(\mathbf{r})\rangle, \tag{2}$$

$$\mathbf{A}_{n,m} = i\hbar\langle\chi_n(\mathbf{r})|\nabla\chi_m(\mathbf{r})\rangle. \tag{3}$$

The off-diagonal elements of the matrices V and $\mathbf{A}$ are typically much smaller than the difference of the dressed atomic energies. In this case an adiabatic approximation can be applied, which amounts to neglecting the off-diagonal contributions. This leads to a separation of the dynamics: Atoms in any one of the dressed states evolve according to a separate Hamiltonian with a $\mathsf{U}(1)$, i.e., Abelian gauge potential.

The adiabatic approximation fails, however, if there are degenerate (or nearly degenerate) dressed states. This is the case we are interested in. Off-diagonal (nonadiabatic) couplings between the degenerate dressed states can then no longer be ignored. Suppose the first q atomic dressed states are degenerate, and these levels are well separated from the remaining $N - q$. Neglecting transitions to the latter, i.e., projecting the full Hamiltonian to this subspace leads to the Schrödinger equation for the reduced column vector $\tilde{\Psi} = (\Psi_1, \ldots, \Psi_q)^\top$

$$i\hbar\frac{\partial}{\partial t}\tilde{\Psi} = \left[\frac{1}{2m}(-i\hbar\nabla - \mathbf{A})^2 + V + \Phi\right]\tilde{\Psi}, \tag{4}$$

where $\mathbf{A}$ and V are the truncated $q \times q$ matrices with elements defined by Eqs. (2) and (3). In addition, a scalar potential arises from this projection, which is again a $q \times q$ matrix,

$$\begin{aligned}\Phi_{n,m} &= \frac{1}{2m}\sum_{l=q+1}^{N}\mathbf{A}_{n,l}\cdot\mathbf{A}_{l,m} \\ &= \frac{\hbar^2}{2m}\left(\langle\nabla\chi_n|\nabla\chi_m\rangle + \sum_{k=1}^{q}\langle\chi_n|\nabla\chi_k\rangle\langle\chi_k|\nabla\chi_m\rangle\right)\end{aligned} \tag{5}$$

with $n, m \in (1, \ldots, q)$. The reduced $q \times q$ matrix $\mathbf{A}$ is called the Berry connection.

Since the adiabatic states $|\chi_1\rangle, \ldots, |\chi_q\rangle$ are degenerate, any basis generated by a local unitary transformation $U(\mathbf{r})$ within the subspace is equivalent. The corresponding local basis change

$$\tilde{\Psi} \rightarrow U(\mathbf{r})\tilde{\Psi} \tag{6}$$

leads to a transformation of the potentials according to

$$\mathbf{A} \rightarrow U(\mathbf{r})\mathbf{A}U^\dagger(\mathbf{r}) - i\hbar(\nabla U(\mathbf{r}))U^\dagger(\mathbf{r}), \tag{7}$$

$$\Phi \rightarrow U(\mathbf{r})\Phi U^\dagger(\mathbf{r}). \tag{8}$$

These transformation rules show the gauge character of the potentials $\mathbf{A}$ and Φ.

The Berry connection or vector potential $\mathbf{A}$ is related to a curvature (an effective "magnetic" field) $\mathbf{B}$ as

$$B_i = \frac{1}{2}\epsilon_{ikl}F_{kl}, \qquad F_{kl} = \partial_k A_l - \partial_l A_k - \frac{i}{\hbar}[A_k, A_l]. \tag{9}$$

Note that the term $\frac{1}{2}\varepsilon_{ikl}[A_k, A_l] = (\mathbf{A} \times \mathbf{A})_i$ does not van-

ish in general, since the vector components of $\mathbf{A}$ do not necessarily commute. In fact, this term reflects the non-Abelian character of the gauge potentials.

The generalized magnetic field transforms under local rotations of the degenerate dressed basis (6) as $\mathbf{B} \to U(\mathbf{r})\mathbf{B}U^\dagger(\mathbf{r})$. Thus, as expected, $\mathbf{B}$ is a true gauge field.

We now construct a scheme of laser-atom interactions that leads to a U(2) gauge potential. The first requirement is the presence of degenerate (or nearly degenerate) dressed states. Such a condition is fulfilled, e.g., for the two systems shown in Fig. 1. Each of them has two degenerate dark states [29], i.e., dressed eigenstates with no component of the excited, radiatively decaying level. Thus, the gauge potentials are 2×2 matrices. A truly non-Abelian situation emerges if the matrices $\{A_x, A_y, A_z, \Phi\}$ do not commute. For this it is necessary that the off-diagonal element $i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_2(\mathbf{r})\rangle$ is nonzero. One can easily check that this expression always vanishes for the system discussed in [14] and shown in Fig. 1(a). It is nonvanishing, however, for the so-called tripod scheme shown in Fig. 1(b) [21].

The Hamiltonian of the tripod system reads in interaction representation as

$$\hat{H}_0 = -\hbar(\Omega_1|0\rangle\langle 1| + \Omega_2|0\rangle\langle 2| + \Omega_3|0\rangle\langle 3|) + \text{H.c.} \quad (10)$$

Parametrizing the Rabi frequencies Ω_μ with angle and phase variables according to $\Omega_1 = \Omega\sin\theta\cos\phi e^{iS_1}$, $\Omega_2 = \Omega\sin\theta\sin\phi e^{iS_2}$, $\Omega_3 = \Omega\cos\theta e^{iS_3}$, where $\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2 + |\Omega_3|^2}$, the adiabatic dark states read

$$|D_1\rangle = \sin\phi e^{iS_{31}}|1\rangle - \cos\phi e^{iS_{32}}|2\rangle, \quad (11)$$

$$|D_2\rangle = \cos\theta\cos\phi e^{iS_{31}}|1\rangle + \cos\theta\sin\phi e^{iS_{32}}|2\rangle - \sin\theta|3\rangle, \quad (12)$$

with $S_{ij} = S_i - S_j$. It is now straightforward to calculate the vector and scalar gauge potentials. This yields

$$\begin{aligned}
\Phi_{11} &= \frac{\hbar^2}{2m}\sin^2\theta\left(\frac{1}{4}\sin^2(2\phi)(\nabla S_{12})^2 + (\nabla\phi)^2\right),\\
\Phi_{12} &= \frac{\hbar^2}{2m}\sin\theta\left(\frac{1}{2}\sin(2\phi)\nabla S_{12} - i\nabla\phi\right)\left(\frac{1}{2}\sin(2\theta)\right.\\
&\quad\left.\times(\cos^2\phi\nabla S_{13} + \sin^2\phi\nabla S_{23}) - i\nabla\theta\right), \quad (13)\\
\Phi_{22} &= \frac{\hbar^2}{2m}\left(\frac{1}{4}\sin^2(2\theta)(\cos^2\phi\nabla S_{13} + \sin^2\phi\nabla S_{23})^2\right.\\
&\quad\left. + (\nabla\theta)^2\right).
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{A}_{11} &= \hbar(\cos^2\phi\nabla S_{23} + \sin^2\phi\nabla S_{13}),\\
\mathbf{A}_{12} &= \hbar\cos\theta[\tfrac{1}{2}\sin(2\phi)\nabla S_{12} - i\nabla\phi],\\
\mathbf{A}_{22} &= \hbar\cos^2\theta(\cos^2\phi\nabla S_{13} + \sin^2\phi\nabla S_{23}), \quad (14)
\end{aligned}$$

Since the level scheme considered in Fig. 1 corresponds to that of alkali atoms where $|1\rangle$, $|2\rangle$, and $|3\rangle$ are Zeeman components of hyperfine levels, it is natural to assume that the external trapping potential is diagonal in these states and has the form $V = V_1(\mathbf{r})|1\rangle\langle 1| + V_2(\mathbf{r})|2\rangle\langle 2| + V_3(\mathbf{r})|3\rangle\langle 3|$. This still takes into account the fact that magnetic, magneto-optical, or optical dipole forces can be different in different Zeeman states. According to Eq. (2), the external potential in the adiabatic basis is then given by a 2×2 matrix with elements $V_{jk} = \langle D_j|V|D_k\rangle$. Using the expressions for the dark states (11) and (12), we arrive at

$$\begin{aligned}
V_{11} &= V_2\cos^2\phi + V_1\sin^2\phi,\\
V_{12} &= \tfrac{1}{2}(V_1 - V_2)\cos\theta\sin(2\phi), \quad (15)\\
V_{22} &= (V_1\cos^2\phi + V_2\sin^2\phi)\cos^2\theta + V_3\sin^2\theta.
\end{aligned}$$

At this point it is instructive to consider some specific examples. Let us first assume that the laser field coupling levels $|1\rangle$ and $|2\rangle$ are copropagating, and have the same frequency and the same orbital angular momentum (if any). In this case their relative phase is fixed and can be set $S_{12} = 0$. This leads to $S_{13} = S_{23} \equiv S$, and the expressions for the vector potential simplify to

$$\mathbf{A} = \hbar\begin{pmatrix} \nabla S & -i\cos\theta\nabla\phi \\ i\cos\theta\nabla\phi & \cos^2\theta\nabla S \end{pmatrix}. \quad (16)$$

The components of the 2×2 matrix of the effective magnetic field can easily be evaluated and read

$$\begin{aligned}
\mathbf{B}_{11} &= 0,\\
\mathbf{B}_{12} &= i\hbar\sin\theta e^{-iS}\nabla\theta \times \nabla\phi - \hbar\cos\theta e^{-iS}\\
&\quad\times \nabla S\nabla\phi(1 + \cos^2\theta),\\
\mathbf{B}_{22} &= -2\hbar\cos\theta\sin\theta\nabla\theta \times \nabla S. \quad (17)
\end{aligned}$$

One recognizes that a large magnetic field requires large gradients of the relative intensities of the fields, parametrized by the angles ϕ and θ and a large gradient of the relative phase S. Gradients of ϕ and θ on the order of the wave number k can be achieved by using standing-wave fields. Large gradients of S can be obtained from a running wave Ω_3 orthogonal to the other two or by a vortex beam with large orbital angular momentum. In this case magnetic fluxes as large as one (in normalized units) can be reached.

We now construct a specific field configuration that leads to a magnetic monopole. For this, let us consider two copropagating and circularly polarized fields $\Omega_{1,2}$ with opposite orbital angular momenta $\pm\hbar$ along the propaga-

tion axis z. The field Ω_3 propagates in the x direction and is linearly polarized along the y axis:

$$\Omega_{1,2} = \Omega_0 \frac{\rho}{R} e^{i(kz \mp \varphi)}, \qquad \Omega_3 = \Omega_0 \frac{z}{R} e^{ik'x}. \tag{18}$$

Here ρ is the distance from the z axis and φ the azimuthal angle around this axis. It should be noted that these fields have a vanishing divergence and obey the Helmholtz equation. This is in contrast to the fields that have been suggested in [30] to create an Abelian magnetic monopole in a Λ system. The total intensity of the laser fields (18) vanishes at an origin, which is a singular point.

The vector potential associated with the fields can be calculated from Eq. (14). It reads

$$\mathbf{A} = -\hbar \frac{\cos\vartheta}{r\sin\vartheta} \hat{e}_\varphi \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{\hbar}{2}(k\hat{e}_z - k'\hat{e}_x) \times \left[(1+\cos^2\vartheta) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + (1-\cos^2\vartheta) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]. \tag{19}$$

The first term proportional to σ_x corresponds to a magnetic monopole of strength one at the origin. This is easily seen by calculating the magnetic field

$$\mathbf{B} = \frac{\hbar}{r^2} \hat{e}_r \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \cdots. \tag{20}$$

The dots indicate nonmonopole field contributions proportional to σ_z, σ_y, and the unity matrix.

In this Letter we have shown that the adiabatic motion of multilevel atoms interacting with spatially varying laser fields in the tripod-coupling configuration can lead to $\mathsf{U}(2)$ non-Abelian gauge potentials. The system can easily be generalized to effective $\mathsf{U}(n)$, $n > 2$, gauge structures using atomic configurations with more than three laser fields coupling to a common excited state. The strength of the effective magnetic fields can be large if standing-wave configurations or light beams with large orbital angular momentum are used. As a specific example, we have identified a configuration of laser fields that leads to a magnetic monopole.

Our approach is complementary to the recent proposal of Osterloh *et al.* [31], who suggested the generation of effective non-Abelian fields in lattice gases. For this, they employed a state-dependent manipulation of tunneling amplitudes by lasers. These proposals make the study of interacting degenerate Bose or Fermi gases in non-Abelian gauge fields experimentally feasible.

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